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3 vol

3401

A TREATISE

ON

REFRIGERATING
AND
ICE-MAKING MACHINERY

PREPARED FOR STUDENTS OF
THE INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

Volume V

ANSWERS TO QUESTIONS

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A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

INCLUDED IN

VOLS. I AND II,

EXCEPT THE

EXAMPLES FOR PRACTICE.

It will be noticed that the Key is divided into sections which correspond to the sections containing the questions and examples at the ends of Vols. I and II. The answers and solutions are so numbered as to be similar to the numbers before the questions to which they refer.

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ARITHMETIC.

(QUESTIONS 1–76.)

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.
605 = Six hundred five.
28,284 = Twenty-eight thousand, two hundred eighty-four.
9,006,042 = Nine millions, six thousand and forty-two.
850,317,002 = Eight hundred fifty millions, three hundred seventeen thousand and two.
700,004 = Seven hundred thousand and four.

(6) Seven thousand six hundred = 7,600.
Eighty-one thousand, four hundred two = 81,402.
Five millions, four thousand and seven = 5,004,007.
One hundred and eight millions, ten thousand and one = 108,010,001.
Eighteen millions and six = 18,000,006.
Thirty thousand and ten = 30,010.

(7) See Art. **71.**

(8) See Art. **76.**

(9) See Art. **73.**

(10) See Art. **73.**

(11) See Art. **74.**

(12) $\frac{13}{8}$ is an improper fraction, since its numerator 13 is greater than its denominator 8.

(13) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(14) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms, we divide both numerator and denominator by 4, and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$.

(15) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6. To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since this denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4, we will have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form*.

(16) In order to reduce a mixed number to an improper fraction, we must *multiply the whole number by the denominator of the fraction*, and *add the numerator of the fraction to that product*. This *result* is the *numerator of the improper fraction*, of which the *denominator is the denominator of the fractional part of the mixed number*.

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$; hence, in

7 there are $7 \times \frac{8}{8} = \frac{56}{8}$. Add the $\frac{7}{8}$ of the mixed number and we obtain $\frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\frac{5}{16} = \frac{(13 \times 16) + 5}{16} = \frac{213}{16};\ 10\frac{3}{4} = \frac{(10 \times 4) + 3}{4} = \frac{43}{4}.$$

(17) The value of a fraction is obtained by dividing the numerator by the denominator.

To obtain the value of the fraction $\frac{13}{2}$ we divide the numerator 13 by the denominator 2. 2 is contained in 13 six times, with 1 remaining. This 1 remaining is written over the denominator 2, thereby making the fraction $\frac{1}{2}$, which is annexed to the whole number 6, and we obtain $6\frac{1}{2}$ as the mixed number. The reason for performing this operation is the following: In 1 there are $\frac{2}{2}$ (two halves), and in $\frac{13}{2}$ (thirteen halves) there are as many ones (1) as 2 is contained times in 13, which is 6, and $\frac{1}{2}$ (one-half) remaining. Hence, $\frac{13}{2} = 6 + \frac{1}{2} = 6\frac{1}{2}$, the required mixed number.

$$\frac{17}{4} = 4\frac{1}{4};\ \frac{69}{16} = 4\frac{5}{16};\ \frac{16}{8} = 2;\ \frac{67}{64} = 1\frac{3}{64}.$$

(18) A fraction is one or more of the equal parts of a unit, and is expressed by a numerator and a denominator, while a decimal fraction is a number of *tenths*, *hundredths*, *thousandths*, etc., of a unit, and is expressed by placing a period (.) called a decimal point, to the left of the figures of the number, and omitting the denominator.

(19) See Arts. **158** and **159.**

(20) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of the quotient, or .5, since only *one* cipher was annexed to the numerator.

$\frac{7}{8}$)7.000
.875

$\frac{65}{100}$)65.00(.65
600
500
500

$\frac{5}{32}$)5.00000(.15625
32
180
160
200
192
80
64
160
160

$\frac{125}{1000}$)125.000(.125
1000
2500
2000
5000
5000

(21)

	tenths.	hundredths.	
.	0	8	= *Eight* **hundredths.**

	tenths.	hundredths.	thousandths.	
.	1	3	1	= *One hundred thirty-one* **thousandths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	
.	0	0	0	1	= *One* **ten-thousandth.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
.	0	0	0	0	2	7	= *Twenty-seven* **millionths.**

tenths.
hundredths.
thousandths.
ten-thousandths.

.0 1 0 8 = *One hundred eight* **ten-thousandths.**

tenths.
hundredths.
thousandths.
ten-thousandths.

93.0 1 0 1=Ninety-three, and *one hundred one* **ten-thousandths.**

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the decimal point towards the right, beginning with tenths, to as many places as there are figures, and the *name* of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example given, the simple reading of the figure is *eight*, and the name of its position in the decimal scale is **hundredths,** so that the decimal reading is *eight* **hundredths.** Similarly, the figures in the fourth example are ordinarily read *twenty-seven;* the name of the position of the figure 7 in the decimal scale is **millionths,** giving, therefore, the decimal reading as *twenty-seven* **millionths.**

If there should be a whole number before the decimal point, read it as you would read any whole number, and read the decimal as you would if the whole number were not there; or, read the whole number and then say "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three, and *one* hundred one ten-thousandths."

(22) See Arts. **136** and **141.**

(23) See Art. **148.**

(24) See Art. **155.**

(25) In adding whole numbers, place the numbers to be added directly under each other, so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place 9 units under the units column and reserve 1 ten for the column of tens, $8 + 7 + 9 + 1 = 25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $4 + 5 + 2 + 2 = 13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $2 + 5 + 3 + 1 = 11$ thousands, or 1 ten-thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten-thousand for the column of ten-thousands. $6 + 1 = 7$ ten-thousands. Place this seven ten-thousands in the ten-thousands column. There is but one figure, 8, in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

$$\begin{array}{r} 3290 \\ 504 \\ 865403 \\ 2074 \\ 81 \\ 7 \\ \hline 871359 \end{array} \quad \text{Ans.}$$

A simpler (though less scientific) explanation of the same problem is the following : $7 + 1 + 4 + 3 + 4 + 0 = 19$; write the nine and reserve the 1. $8 + 7 + 0 + 0 + 9 + 1$ reserved $= 25$; write the 5 and reserve the 2. $0 + 4 + 5 + 2 + 2$ reserved $= 13$; write the 3 and reserve the 1. $2 + 5 + 3 + 1$ reserved $= 11$; write the 1 and reserve 1. $6 + 1$ reserved $= 7$; write the 7. Bring down the 8 to its place in the sum.

(26)

$$\begin{array}{r} 709 \\ 8\,304\,725 \\ 391 \\ 100\,302 \\ 300 \\ 909 \\ \hline 8{,}407{,}336 \end{array} \quad \text{Ans.}$$

(27) $\frac{1}{8}+\frac{2}{8}+\frac{5}{8}=\frac{1+2+5}{8}=\frac{8}{8}=1.$ Ans.

When the *denominators* of the fractions to be added *are alike*, we know that the units are divided into the *same number of parts* (in this case *eighths*); we, therefore, *add the numerators* of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$, or 1, as the sum.

(28) When the *denominators* are *not alike* we know that the units are divided into *unequal parts*, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators, and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all of these fractions to 16ths and then add their numerators.

$\frac{1}{4}+\frac{3}{8}+\frac{5}{16}=?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms of the fraction by some number which will make the denominator 16. This number evidently is 4, hence, $\frac{1\times 4}{4\times 4}=\frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3\times 2}{8\times 2}=\frac{6}{16}$. The fractions now have a common denominator 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16}+\frac{6}{16}+\frac{5}{16}=\frac{4+6+5}{16}=\frac{15}{16}$. Ans.

(29) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. Adding the two fractional parts of the mixed numbers, we have $\frac{10}{16} + \frac{7}{16} = \frac{10 + 7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42 + 31 + 9 + 1\frac{1}{16} = ?$

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

$$\begin{array}{r} 42 \\ 31 \\ 9 \\ 1\frac{1}{16} \\ \hline 83\frac{1}{16} \end{array} \text{ Ans.}$$

(30) $29\frac{3}{4} + 50\frac{5}{8} + 41 + 69\frac{3}{16} = ?$ $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}$; $\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. $\frac{12}{16} + \frac{10}{16} + \frac{3}{16} = \frac{12 + 10 + 3}{16} = \frac{25}{16} = 1\frac{9}{16}$.

The problem now becomes $29 + 50 + 41 + 69 + 1\frac{9}{16} = ?$

$$\begin{array}{rl} 29 & \text{square inches.} \\ 50 & \text{square inches.} \\ 41 & \text{square inches.} \\ 69 & \text{square inches.} \\ 1\frac{9}{16} & \text{square inches.} \\ \hline 190\frac{9}{16} & \text{square inches. Ans.} \end{array}$$

(31) In addition of decimals, the *decimal points must be placed directly under each other*, so that *tenths* will come *under tenths*, *hundredths under hundredths*, *thousandths under thousandths*, etc. The addition is then performed as in whole numbers, *the decimal point of the sum being placed directly under the decimal points above.*

$$\begin{array}{l} .125 \\ .7 \\ .089 \\ .4005 \\ .9 \\ .000027 \\ \hline 2.214527 \end{array} \quad \text{Ans.}$$

(32)

$$\begin{array}{r@{.}l} 927&416 \\ 8&274 \\ 372&6 \\ 62&07938 \\ \hline 1370&36938 \end{array} \quad \text{Ans.}$$

(33)

tenths.
hundredths.
thousandths.
ten-thousandths.
hundred-thousandths.
millionths.

$$\begin{array}{l} .017 \\ .2 \\ .000047 \\ \hline .217047 \end{array}$$

.217047 = *Two hundred and seventeen thousand and forty-seven* **millionths.** Ans.

(34) It will be more convenient to add these numbers together if we express $348\frac{3}{4}$ decimally, since the remaining numbers are decimals. $348\frac{3}{4}$ is equal to 348.75, since $\frac{3}{4}=.75$.

$$\begin{array}{l} 4)3.00(.75 \\ \;\;\underline{28} \\ \;\;\;\;20 \\ \;\;\;\;\underline{20} \end{array}$$

Since we now have the different weights expressed decimally, we find the total weight of the four lengths of shafting to be 316.5 lb. + 402.3 lb. + 348.75 lb. + 309.4 lb., or 1,376.95 lb. Ans.

316.5 lb.
402.3 lb.
348.75 lb.
309.4 lb.
1376.95 lb. Ans.

(35) If the cost of the coal consumed by a nest of steam boilers amounts to $15.83 on Monday, to $14.70 on Tuesday, to $14.28 on Wednesday, to $13.87 on Thursday, to $14.98 on Friday, and to $12.65 on Saturday, then, we find the total cost of the week's supply by adding the different amounts together; hence, $15.83 + $14.70 + $14.28 + $13.87 + $14.98 + $12.65 = $86.31.

$15.83
14.70
14.28
13.87
14.98
12.65
$86.31 Ans.

(36) The steam engine, during the 12-hour test, showed that the number of revolutions made were 150,508, since 12,600 + 12,444 + 12,467 + 12,528 + 12,468 + 12,590 + 12,610 + 12,589 + 12,576 + 12,558 + 12,546 + 12,532 = 150,508 rev. Ans.

12600 revolutions.
12444 revolutions.
12467 revolutions.
12528 revolutions.
12468 revolutions.
12590 revolutions.
12610 revolutions.
12589 revolutions.
12576 revolutions.
12558 revolutions.
12546 revolutions.
12532 revolutions.
150508 revolutions. Ans.

(37) In subtracting whole numbers, place the subtrahend, or smaller number, under the minuend, or larger number, so that the right-hand figures stand directly under each other. Begin *at the right* to subtract. We can not subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. 1 *ten* = 10 *units*, so we have 10 units + 2 units = 12 units. Then, 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so only 5 tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We can not subtract 3 thousands from 0 thousands, so we take 1 ten-thousand from 5 ten-thousands and add it to the 0 thousands. 1 *ten-thousand* = 10 *thousands*, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten-thousand from 5 ten-thousands and have 4 ten-thousands remaining. Since there are no ten-thousands in the subtrahend, the 4 in the ten-thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

$$(a)\quad \begin{array}{r} 50962 \\ 3338 \\ \hline 47624 \end{array} \text{ Ans.}$$

$$(b)\quad \begin{array}{r} 15339 \\ 10001 \\ \hline 5338 \end{array} \text{ Ans.}$$

(38) $(a)\quad \begin{array}{r} 70968 \\ 32975 \\ \hline 37993 \end{array} \text{ Ans.} \qquad (b)\quad \begin{array}{r} 100000 \\ 98735 \\ \hline 1265 \end{array} \text{ Ans.}$

(39) $\frac{7}{8} - \frac{7}{16} = ?$ When the *denominators* of fractions are *not alike*, it is evident that the units are divided into *unequal parts*, therefore, before subtracting, *reduce the fractions to fractions having a common denominator*. Then, *subtract the numerators, and place the remainder over the common denominator.*

$\frac{7\times 2}{8\times 2}=\frac{14}{16}$. $\frac{14}{16}-\frac{7}{16}=\frac{14-7}{16}=\frac{7}{16}$. Ans.

$13-7\frac{7}{16}=?$ This problem may be solved in two ways:

First: $13=12\frac{16}{16}$, since $\frac{16}{16}=1$, and $12\frac{16}{16}=12+\frac{16}{16}=12+1=13$.

$$\begin{array}{r} 12\frac{16}{16} \\ 7\frac{7}{16} \\ \hline 5\frac{9}{16} \end{array}$$

We can now subtract the whole numbers separately, and the fractions separately, and obtain $12-7=5$ and $\frac{16}{16}-\frac{7}{16}=\frac{16-7}{16}=\frac{9}{16}$. $5+\frac{9}{16}=5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$13=\frac{13}{1}=\frac{13\times 16}{1\times 16}=\frac{208}{16}$. $7\frac{7}{16}=\frac{(7\times 16)+7}{16}=\frac{112+7}{16}=\frac{119}{16}$.

Subtracting, we have $\frac{208}{16}-\frac{119}{16}=\frac{208-119}{16}=\frac{89}{16}$ and

$$\frac{89}{16}=\begin{array}{l} 16)89(5\frac{9}{16} \\ 80 \\ \hline 9 \\ \hline 16 \end{array}$$

the same result that was obtained by the first method.

$312\frac{9}{16}-229\frac{5}{32}=?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this, we have $\frac{9}{16}=\frac{9\times 2}{16\times 2}=\frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312-229=83$ and $\frac{18}{32}-\frac{5}{32}=\frac{18-5}{32}=\frac{13}{32}$.

$$\begin{array}{r} 312\frac{18}{32} \\ 229\frac{5}{32} \\ \hline 83\frac{13}{32} \end{array} \quad 83+\frac{13}{32}=83\frac{13}{32}. \text{ Ans.}$$

(40) (*a*) In subtraction of decimals, *place the decimal points directly under each other*, and proceed as in the subtraction of whole numbers, placing the *decimal point in the remainder directly under the decimal points above.*

$$\begin{array}{r} 709.6300 \\ .8514 \\ \hline 708.7786 \end{array} \text{ Ans.}$$

In the above example, we proceed as follows: We can not subtract 4 ten-thousandths from 0 ten-thousandths, and as there are no thousandths, we take 1 hundredth from the three hundredths. 1 *hundredth* = 10 *thousandths* = 100 *ten-thousandths.* 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 *thousandths* + 6 *ten-thousandths.* Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths leaves but 2 hundredths now.) Since we can not do this, we take 1 tenth from 6 tenths. 1 tenth = 10 hundredths, and 10 hundredths + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this can not be done, we take 1 unit from the 9 units. 1 *unit* = 10 *tenths.* 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (one unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units, hence, we write 708 in the remainder.

$$(b)\ \begin{array}{r} 81.963 \\ 1.700 \\ \hline 80.263 \end{array}\ \text{Ans.} \qquad (c)\ \begin{array}{r} 18.00 \\ .18 \\ \hline 17.82 \end{array}\ \text{Ans.} \qquad (d)\ \begin{array}{r} 1.000 \\ .001 \\ \hline .999 \end{array}\ \text{Ans.}$$

(*e*) 872.1 − (.8721 + .008) = ? In this problem we are to subtract (.8721 + .008) from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

$$\begin{array}{r} .8721 \\ .0080 \\ \hline .8801 \end{array}\ \textit{sum.}$$

fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction. First, we will reduce the fractional parts $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to equivalent fractions having the least common denominator. In this case the least common denominator equals the product of the denominators 5, 3, and 4, since we can not divide any two of them by any number (except 1) without having a remainder, as can be done in the examples in Art. **96.** Hence, the least common denominator $= 5 \times 3 \times 4 = 60$. Reducing $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to fractions having this least common denominator, we have 60 divided by the first denominator 5, equals 12. Then, $\frac{4}{5} \times \frac{12}{12} = \frac{48}{60}$. 60 divided by the second denominator 3, equals 20. Then, $\frac{1}{3} \times \frac{20}{20} = \frac{20}{60}$. 60 divided by the third denominator 4, equals 15. Then, $\frac{3}{4} \times \frac{15}{15} = \frac{45}{60}$. The sum of these fractions equals

$$\frac{48}{60} + \frac{20}{60} + \frac{45}{60} = \frac{48 + 20 + 45}{60} = \frac{113}{60}, \text{ or } 1\frac{53}{60}.$$

The problem now becomes $482 + 316 + 390 + 1\frac{53}{60}$, the sum of which equals $1{,}189\frac{53}{60}$.

$$\begin{array}{r} 482 \\ 316 \\ 390 \\ 1\frac{53}{60} \\ \hline 1189\frac{53}{60} \end{array}$$

1500 represents the actual horsepower required.

$1189\frac{53}{60}$ represents the indicated horsepower of the engines in use.

$310\frac{7}{60}$, or $310.11\frac{2}{3}$ = the H. P. to be developed by the new engine. Ans.

$\frac{7}{60}$ reduced to its equivalent decimal $= \frac{7}{60)}\,7.00(.11\frac{2}{3}$

$$\begin{array}{r}60\\ \hline 100\\ 60\\ \hline \frac{4\cancel{0}}{6\cancel{0}} = \frac{2}{3}.\end{array}$$

(43)

3040 = No. of gallons in the tank at the beginning of the day.
4780 = No. of gallons pumped in during the morning.
7820 = No. of gallons in the tank after 4,780 gallons were added.
7240 = No. of gallons drawn out during the morning.
580 = No. of gallons in the tank at the beginning of the afternoon.
8675 = No. of gallons pumped in during the afternoon.
9255 = No. of gallons in the tank after 8,675 gallons were added.
7633 = No. of gallons drawn out during the afternoon.
1622 = No. of gallons remaining in the tank at night. Ans.

(44) The height of the stack equals 45 feet. Since six of the plates which make this stack are 7 feet long, the length of the six plates equals 6 × 7 or 42 feet. However, there is an allowance of 15 inches, or 1 foot 3 inches, for lapping, which, deducted from 42 feet, makes a total length of 40 feet 9 inches. Since the total height of the stack is 45 feet, the length of the seventh plate must equal 45 feet—40 feet 9 inches, or 4 ft. 3 in. Ans.

45 feet 0 inches
40 feet 9 inches
4 feet 3 inches

(45) Since the inside diameter of the steam pipe is 6.06 inches, and the outside diameter is 6.62 inches, there is a difference of 6.62 — 6.06, or .56 of an inch, in both diameters.

But .56 of an inch is just twice the thickness of the pipe; hence, the pipe is $\frac{1}{2}$ of .56, or .28 of an inch thick.

(46) In the multiplication of whole numbers, place the multiplier under the multiplicand, and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

$$\text{(}a\text{)}\quad \begin{array}{r} 526{,}387 \\ 7 \\ \hline 3{,}684{,}709 \end{array} \quad \text{Ans.}$$

7 times 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 + 4 tens reserved = 60 tens, or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7 × 3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve the 2 thousands. 7 × 6 thousands = 42 thousands; 42 + 2 thousands reserved = 44 thousands, or 4 ten-thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten thousands. 7 × 2 ten-thousands = 14 ten-thousands; 14 + 4 ten-thousands reserved = 18 ten-thousands, or 1 hundred-thousand and 8 ten-thousands. Write the 8 ten-thousands and reserve the 1 hundred-thousand. 7 × 5 hundred-thousands = 35 hundred-thousands; 35 + 1 hundred-thousand reserved = 36 hundred-thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred-thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times 7 = 49; write the 9 and reserve the 4. 7 times 8 = 56; 56 + 4 reserved = 60; write the 0 and reserve the 6. 7 times 3 = 21; 21 + 6 reserved = 27; write the 7 and reserve the 2. 7 × 6 = 42; 42 + 2 reserved = 44; write the 4 and reserve 4. 7 × 2 = 14; 14 + 4 reserved = 18; write the 8 and reserve 1. 7 × 5 = 35; 35 + 1 reserved = 36; write the 36.

(*b*)

$$\begin{array}{r} 700{,}298 \\ 17 \\ \hline 4902086 \\ 700298 \\ \hline 11{,}905{,}066 \end{array} \text{ Ans.}$$

In this case the multiplier is 17 *units*, or 1 *ten* and 7 *units*, so that the product is obtained by adding two partial products, namely, 7 × 700,298 and 10 × 700,298. The actual operation is performed as follows:

7 times 8 = 56; write the 6 and reserve the 5. 7 times 9 = 63; 63 + 5 reserved = 68; write the 8 and reserve the 6. 7 times 2 = 14; 14 + 6 reserved = 20; write the 0 and reserve the 2. 7 times 0 = 0; 0 + 2 reserved = 2; write the 2. 7 times 0 = 0; 0 + 0 reserved = 0; write the 0. 7 times 7 = 49; 49 + 0 reserved = 49; write the 49.

To multiply by the 1 ten we have 1 ten times 8 units = 8 tens and 0 units. We do not write the 0 units, but write the 8 tens under the 8 tens in the first partial product above. We next multiply 1 ten, and 9 tens = 90 tens = 9 hundreds + 0 tens; write the 9 hundreds under the 0 hundreds of the first partial product above. Again, 1 ten times 2 hundreds = 2 thousands; write the 2 thousands under the 2 thousands above. 1 ten times 0 thousands = 0 ten-thousands; write the 0 in the ten-thousands place. 1 ten times 0 ten-thousands = 0 hundred-thousands; write the 0 in the hundred-thousands place. 1 ten times 7 hundred-thousands = 7 millions; write the 7 in the millions place. This completes the second partial product. Add the two partial products; their sum equals the entire product.

(*c*)

$$\begin{array}{r} 217 \\ 103 \\ \hline 651 \\ 000 \\ 217 \\ \hline 22351 \\ 67 \\ \hline 156457 \\ 134106 \\ \hline 1{,}497{,}517 \end{array} \text{ Ans.}$$

Multiply any two of the numbers together, and multiply their product by the third number.

(**47**) If your watch ticks every second, to find how many times it ticks in one week it is necessary to find the number of seconds in 1 week.

60 seconds = 1 minute.
60 minutes = 1 hour.
3,600 seconds = 1 hour.
24 hours = 1 day.
14400
7200
86,400 seconds = 1 day.
7 days = 1 week.
604,800 seconds in one week, or the number of times that
Ans. your watch ticks in a week.

(**48**) (*a*)
.107
.013
321
107
.001391 Ans.

There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3 or 6 decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary six decimal places.

(*b*)
203
2.03
609
000
406
412.09
.203
123627
00000
82418
83.65427 Ans.

There are 2 decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0 or 2 decimal places in the first product.

Since in the second multiplication there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2 or 5 decimal places in the second product.

When there are one or more ciphers in the multiplier, multiply just the same as with the other figures.

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parentheses, and then whatever may be required by the sign between the parentheses.

$$\begin{array}{r} 31.85 \\ 2.7 \\ \hline 22295 \\ 6370 \\ \hline 85.995 \end{array}$$

The first parenthesis shows that the numbers 2.7 and 31.85 are to be multiplied together.

The second parenthesis shows that .316 is to be taken from 3.16.

$$\begin{array}{r} 3.160 \\ .316 \\ \hline 2.844 \end{array}$$

The product obtained by performing the operation indicated by the signs within the first parenthesis is now multiplied by the remainder obtained by performing the operation indicated by the signs within the second parenthesis.

$$\begin{array}{rl} 85.995 & \\ 2.844 & \\ \hline 343980 & \\ 343980 & \\ 687960 & \\ 171990 & \\ \hline 244.569780 & \text{Ans.} \end{array}$$

(*d*) $(107.8 + 6.541 - 31.96) \times 1.742 = ?$

$$\begin{array}{rr} & 107.8 \\ + & 6.541 \\ \hline & 114.341 \\ - & 31.96 \\ \hline & 82.381 \end{array} \qquad \begin{array}{rrl} & 82.381 & \\ \times & 1.742 & \\ \hline & 164762 & \\ & 329524 & \\ & 576667 & \\ & 82381 & \\ \hline & 143.507702 & \text{Ans.} \end{array}$$

(49) If an engine and boiler are worth \$3,246, and the building is worth 3 times as much plus \$1,200, then the building is worth

$$\begin{array}{rrl} & \$3246 & \\ & 3 & \text{times} \\ \hline & 9738 & \\ \text{plus} & 1200 & \\ \hline & \$10938 & = \text{value of building.} \end{array}$$

If the tools are worth twice as much as the building, plus \$1,875, then the tools are worth

$$\begin{array}{rrl} & \$10938 & \\ & 2 & \\ \hline & 21876 & \\ \text{plus} & 1875 & \\ \hline & \$23751 & = \text{value of tools.} \\ \text{Value of building} = & \$10938 & \\ \text{Value of tools} \quad = & 23751 & \\ \hline & \$34689 & = \text{value of the building and tools. } (a) \text{ Ans.} \\ \text{Value of engine and boiler} = & \$3246 & \\ \text{Value of building and tools} = & 34689 & \\ \hline & \$37935 & = \text{value of the whole plant. } (b) \text{ Ans.} \end{array}$$

(50) If one 3-inch tube measures 15½ ft. in length, 60 of these tubes would measure 60 × 15½ ft., or 930 ft. in length. If 1 foot of tubing heats a surface of .728 sq. ft., then it is evident that 930 ft. of tubing would heat a surface of 930 × .728 ft., or 677.04 sq. ft.

$$\begin{array}{r} 15\frac{1}{2} \\ 60 \\ \hline 900 \\ 30 \\ \hline 930 \text{ ft.} \end{array}$$

$$\begin{array}{r} .728 \\ 930 \\ \hline 21840 \\ 6552 \\ \hline 677.040 \text{ sq. ft.} \end{array}$$

(51) If in 1 hour 10 pounds of coal are burned for every square foot of grate area, and 9 pounds of water are evaporated for every pound of coal burned, then in 1 hour there would be 9 × 10 or 90 pounds of water evaporated for 1 sq. ft. of grate area; and since the grate area is 30 sq. ft., the amount of water evaporated would be 30 × 90 = 2,700 lb. Since 2,700 lb. of water are evaporated in 1 hour, in a day of 10 hours there would be 10 × 2,700 lb., or 27,000 lb. of water evaporated.

(52) Each stroke of the engine is 18 inches in length. Since the piston makes 2 strokes for each revolution, it would pass over a distance of 2 × 18 inches = 36 inches, or 3 feet in 1 minute, and in making 160 revolutions it would pass over 160 × 3, or 480 ft. Since 480 ft. are passed over in 1 minute, in 1 hour, or 60 minutes, the distance passed over equals 60 × 480 = 28,800 ft. Since the steam engine runs 6 days a week and $8\frac{1}{2}$ hours per day, the total number of hours it runs per week = 6 × $8\frac{1}{2}$, or 51 hours. If the piston passes over a distance of 28,800 ft. in 1 hour, in 51 hours it would pass over 51 × 28,800 ft., or 1,468,800 ft. Ans.

$$\begin{array}{r} 480\text{ ft.} \\ \times\ 60 \\ \hline 28800\text{ ft.} \end{array}$$

$$\begin{array}{r} 28800 \\ \times\ \ \ 51 \\ \hline 28800 \\ 144000\ \ \\ \hline 1468800 \end{array}$$

(53) Since the first pump delivers $1\frac{1}{4}$ gallons for each stroke, and runs at the rate of 75 strokes per minute, then the number of gallons delivered by the first pump equals 75 × $1\frac{1}{4}$ = 75 × 1.25 = 93.75 gallons.

$$\begin{array}{r} 1.25 \\ \times\ \ \ 75 \\ \hline 625 \\ 875\ \ \\ \hline 93.75 \end{array}$$

$$\frac{7}{8} = .875, \text{ since } \begin{array}{r} 8\,\overline{)\,7.000} \\ .875 \end{array}$$

The second pump delivers $\frac{7}{8}$ or .875 of a gallon for each stroke, and runs at the rate of 115 strokes per minute; then the total number of gallons delivered by the second pump equals 115 × .875 = 100.625 gallons.

$$\begin{array}{r} .875 \\ \times\ 115 \\ \hline 4375 \\ 875\ \ \\ 875\ \ \ \ \\ \hline 100.625 \end{array}$$

```
8)5.000
   .625

  1.625
×    96
  -----
   9750
 14625
 -------
 156.000
```

The third pump delivers $1\frac{5}{8}$, or 1.625 (since $\frac{5}{8} = .625$) gallons, for each stroke, and runs at the rate of 96 strokes per minute; then, the total number of gallons delivered by the third pump equals 96 × 1.625 = 156 gallons.

```
 350.375
      60
---------
21022.500
```

The total number of gallons delivered by the three pumps in one minute equals 93.75 + 100.625 + 156 gallons, or 350.375 gallons. Since there are 60 minutes in 1 hour, we find that the total number of gallons of water fed to the boilers in 1 hour equals 60 × 350.375 = 21,022.5. Ans.

(54) (*a*)

```
84)962842.0000(11462.4047+ Ans.
   84
   --
   122
    84
   ---
    388
    336
    ---
     524
     504
     ---
      202
      168
      ---
       340
       336
       ---
        400
        336
        ---
         640
         588
         ---
          52
```

84 is contained in 96, once. Place 1 as the first figure in the quotient and multiply the divisor 84 by it. Subtract

the product, which is 84, from 96, leaving a remainder of 12. Bring down the next figure in the dividend, which is 2, annex it to 12, making a new dividend of 122.

84 is contained in 122 once. Place 1 as the second figure in the quotient and multiply the divisor 84 by it. Subtract the product (84) from 122, leaving a remainder of 38. Bring down the next figure in the dividend, which is 8, annex it to 38, making a new dividend of 388.

84 is contained in 388 4 times. Place 4 as the third figure in the quotient and multiply the divisor 84 by it. The product is 336. Subtract the product from 388, leaving a remainder of 52. Bring down the next figure, which is 4, annex it to 52, making a new dividend of 524.

84 is contained in 524 6 times. Place 6 as the fourth figure in the quotient. Multiply the divisor 84 by it and subtract the product (504) from 524, leaving a remainder of 20. Bring down the next figure, which is 2, annex it to 20. making a new dividend of 202.

84 is contained in 202 2 times. Place 2 as the fifth figure in the quotient. Multiply the divisor 84 by it, and subtract the product (168) from 202, leaving a remainder of 34. If it is desired to carry the quotient to 4 decimal places, annex 4 ciphers to the dividend and continue in the same way. In the quotient point off as many decimal places as there are ciphers annexed, or, in this case, four decimal places.

```
(b)  63)39728.000(630.603+  Ans.
        378
        ---
         192
         189
         ----
           380*
           378
           ----
             200
             189
             ---
              11
```

*63 is not contained in 38, so we place a cipher in the quotient and bring down the next figure in the dividend, which is a cipher that has been annexed, making a new dividend of 380.

63 is contained in 380 6 times. 6×63=378. Subtracting

378 from 380 leaves 2. Bringing down the next figure in the dividend we have 20 for a new dividend. 63 is not contained in 20, so we place a cipher in the quotient and bring down the next cipher in the dividend, making a new dividend of 200. 63 is contained 3 times in 200.

```
(c) 108)29714.0000(275.1296  Ans.
        216
        ---
        811    (d) 135)406089.0000(3008.0666
        756            405                  Ans.
        ----           ------
         554           *1089
         540            1080
         ----           ------
          140              900
          108              810
          ----             ----
           320              900
           216              810
           ----             ----
           1040              900
            972              810
            ----             ---
             680              90
             648
             ---
              32
```

*135 is not contained in 10, so we place a cipher as the second figure in the quotient. Bringing down the next figure 8 and annexing it to 10, we have a new dividend of 108. 135 is not contained in 108, so we place a cipher as the third figure in the quotient, and bring down the next figure in the dividend, or 9, and annex it to 108, making a new dividend of 1,089. 135 is contained in 1,089 8 times. Write 8 as the fourth figure in the quotient. Multiply 135 by 8 and subtract the product (1,080) from 1,089, which leaves a remainder of 9. Bring down the next figure in the dividend, which is a cipher that has been annexed, annex it to the remainder 9, making a new dividend of 90. As 135 is not contained in 90, we place a 0 in the quotient and bring down another cipher from the dividend, making a new dividend of 900. 135 is contained in 900 6 times.

Write 6 as the next figure in the quotient, multiply 135 by 6 and subtract the product (810) from 900, which leaves a remainder of 90. Bring down the next figure (0) in the dividend, annex it to the remainder 90, making a new dividend of 900. 135 is contained in 900 six times. Place 6 as the next figure in the quotient and multiply the divisor by it. It is plain that each succeeding figure of the quotient will be 6. Point off *four* decimal places in the quotient, since four ciphers were annexed.

(55) In division of fractions, *invert the divisor* (or, in other words, turn it upside down) *and proceed as in multiplication.*

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112.$ Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}.$ Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}.$ Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \frac{113 \times 16}{64 \times 7} = \frac{1,808}{448} =$

$$\frac{452}{112} = \frac{113}{28} \quad \begin{array}{r} 28\,)\,113\,(\,4\tfrac{1}{28}. \\ \underline{112}\phantom{\,(\,4\tfrac{1}{28}.} \\ 1\phantom{\,(\,4\tfrac{1}{28}.} \end{array}$$ Ans.

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both of the mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4) + 3}{4} = \frac{60 + 3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8) + 3}{8} = \frac{32 + 3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor and multiply; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{63 \times 8}{4 \times 35} = \frac{504}{140} = \frac{252}{70} = \frac{126}{35} = \frac{18}{5}.$

$$\frac{18}{5}\quad \begin{array}{r} 5\,)\,18\,(\,3\tfrac{3}{5}. \\ \underline{15}\phantom{\,(\,3\tfrac{3}{5}.} \\ 3\phantom{\,(\,3\tfrac{3}{5}.} \end{array}$$ Ans.

(56) (*a*) $.875 \div \frac{1}{2} = .875 \div .5 \left(\text{since } \frac{1}{2} = .5\right) = 1.75$. Ans. Another way of solving this is to reduce .875 to its equivalent common fraction and then divide.

$.875 = \frac{7}{8}$, since $.875 = \frac{875}{1{,}000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; then, $\frac{7}{8} \div \frac{1}{2} = \frac{7}{8} \times \frac{2}{1} = \frac{7 \times 2}{8} = \frac{14}{8} = \frac{7}{4} = 1\frac{3}{4}$.

Since $\frac{3}{4} = 4\overline{)3.00}(.75$, $1\frac{3}{4} = 1.75$, the same answer as above.

$$\begin{array}{l} \,2\,8 \\ \hline \,2\,0 \\ \,2\,0 \\ \hline \end{array}$$

(*b*) $\frac{7}{8} \div .5 = \frac{7}{8} \div \frac{1}{2} \left(\text{since } .5 = \frac{1}{2}\right) = \frac{7}{8} \times \frac{2}{1} = \frac{7 \times 2}{8} = \frac{14}{8} = \frac{7}{4} = 1\frac{3}{4}$, or 1.75. Ans.

This can also be solved by reducing $\frac{7}{8}$ to its equivalent decimal and dividing by .5; $\frac{7}{8} = .875$; $.875 \div .5 = 1.75$. Since there are three decimal places in the dividend and one in the divisor, there are $3 - 1$, or 2 decimal places in the quotient.

(*c*) $\frac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125} = ?$ We will solve this problem by first reducing the decimals to their equivalent common fractions.

$.375 = \frac{375}{1{,}000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$. $\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$, or the value of the numerator of the fraction.

$.125 = \frac{125}{1{,}000} = \frac{25}{200} = \frac{1}{8}$. Reducing $\frac{1}{8}$ to 16ths, we have $\frac{1 \times 2}{8 \times 2} = \frac{2}{16}$. Then, $\frac{5}{16} - \frac{2}{16} = \frac{3}{16}$, or the value of the denominator of the fraction. The problem is now reduced to

$$\frac{\frac{3}{32}}{\frac{3}{16}} = ? \quad \frac{\frac{3}{32}}{\frac{3}{16}} = \frac{3}{32} \div \frac{3}{16} = \frac{3}{32} \times \frac{16}{3} = \frac{48}{96} = \frac{1}{2} \text{ or } .5. \quad \text{Ans.}$$

(**57**) (*a*) (72 × 48 × 28 × 5) ÷ (84 × 15 × 7 × 6).

Placing the numerator over the denominator the problem becomes

$$\frac{72 \times 48 \times 28 \times 5}{84 \times 15 \times 7 \times 6} = ?$$

The 5 in the numerator and 15 in the denominator are both *divisible* by 5, since 5 divided by 5 equals 1, and 15 divided by 5 equals 3. *Cross off* the 5 and write the 1 *over* it; also *cross off* the 15 and write the 3 *under* it. Thus,

$$\frac{72 \times 48 \times 28 \times \overset{1}{\cancel{5}}}{84 \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

72 in the numerator and 84 in the denominator are *divisible* by 12, since 72 divided by 12 equals 6, and 84 divided by 12 equals 7. *Cross off* the 72 and write the 6 *over* it; also, *cross off* the 84 and write the 7 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times 28 \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

Again, 28 in the numerator and 7 in the denominator are *divisible* by 7, since 28 divided by 7 equals 4, and 7 divided by 7 equals 1. *Cross off* the 28 and write the 4 *over* it; also, *cross off* the 7 and write the 1 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{\cancel{7}}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \underset{1}{\cancel{7}} \times 6} =$$

Again, 48 in the numerator and 6 in the denominator are *divisible* by 6, since 48 divided by 6 equals 8, and 6 divided by 6 equals 1. *Cross off* the 48 and write the 8 *over* it; also, *cross off* the 6 and write the 1 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{6}}} =$$

Again, 6 in the numerator and 3 in the denominator are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and write the 2 *over* it; also, *cross off* the 3 and write the 1 *under* it. Thus

$$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{6}}}.$$

Since there are *no two remaining numbers* (one in the numerator and one in the denominator) *divisible* by *any number* except 1, without a remainder, it is *impossible* to cancel further.

Multiply all the *uncanceled numbers* in the numerator together, and divide their *product* by the *product* of all the *uncanceled* numbers in the denominator. The *result* will be the *quotient.* The *product* of all the *uncanceled numbers* in the numerator equals $2 \times 8 \times 4 \times 1 = 64$; the product of all the *uncanceled* numbers in the denominator equals $7 \times 1 \times 1 \times 1 = 7$.

Hence, $$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{6}}} = \frac{2 \times 8 \times 4 \times 1}{7 \times 1 \times 1 \times 1} = \frac{64}{7} = 9\frac{1}{7}. \text{ Ans.}$$

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes
$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the numerator and 70 in the denominator are both *divisible* by 10, since 50 divided by 10 equals 5, and 70 divided by 10 equals 7. *Cross off* the 50 and write the 5 *over* it; also, *cross off* the 70 and write the 7 *under* it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

Also, 80 in the numerator and 20 in the denominator are *divisible* by 20, since 80 divided by 20 equals 4, and 20 divided by 20 equals 1. *Cross off* the 80 and write the 4 *over* it; also, cross off the 20 and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times \underset{1}{\cancel{20}}} =$$

Again, 16 in the numerator and 24 in the denominator are *divisible* by 8, since 16 divided by 8 equals 2, and 24 divided by 8 equals 3. *Cross off* the 16 and write the 2 *over* it; also, cross off the 24 and write the 3 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times 14}{\underset{7}{\cancel{70}} \times 50 \times \underset{3}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

Again, 60 in the numerator and 50 in the denominator are *divisible* by 10, since 60 divided by 10 equals 6, and 50 divided by 10 equals 5. *Cross off* the 60 and write the 6 *over* it; also, cross off the 50 and write the 5 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times 14}{\underset{7}{\cancel{70}} \times \underset{5}{\cancel{50}} \times \underset{3}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

The 14 in the numerator and 7 in the denominator are *divisible* by 7, since 14 divided by 7 equals 2, and 7 divided by 7 equals 1. *Cross off* the 14 and write the 2 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{5}{\cancel{50}} \times \underset{3}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

The 5 in the numerator and 5 in the denominator are *divisible* by 5, since 5 divided by 5 equals 1. *Cross off* the

5 of the *dividend* and write the 1 *over* it; also, cross off the 5 of the *divisor* and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{\overset{1}{\cancel{5}}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{\underset{1}{\cancel{5}}}{\cancel{50}} \times \underset{\underset{1}{3}}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

The 6 in the numerator and 3 in the denominator are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and place 2 *over* it; also, cross off the 3 and place 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{60}} \times \overset{\overset{1}{\cancel{5}}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{\underset{1}{\cancel{5}}}{\cancel{50}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

Hence, $\dfrac{\overset{4}{\cancel{80}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{60}} \times \overset{\overset{1}{\cancel{5}}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{\underset{1}{\cancel{5}}}{\cancel{50}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{24}} \times \underset{1}{\cancel{20}}} = \dfrac{4 \times 2 \times 1 \times 2 \times 2}{1 \times 1 \times 1 \times 1} = \dfrac{32}{1} = 32.$ Ans.

(58) (*a*) $\dfrac{7}{\dfrac{3}{16}} = 7 \div \dfrac{3}{16} = 7 \times \dfrac{16}{3} = \dfrac{7 \times 16}{3} = \dfrac{112}{3} = 37\dfrac{1}{3}.$ Ans.

The heavy line indicates that 7 is to be divided by $\frac{3}{16}$.

(*b*) $\dfrac{\dfrac{15}{32}}{\dfrac{5}{8}} = \dfrac{15}{32} \div \dfrac{5}{8} = \dfrac{15}{32} \times \dfrac{8}{5} = \dfrac{\overset{3}{\cancel{15}} \times \cancel{8}}{\underset{4}{\cancel{32}} \times \cancel{5}} = \dfrac{3}{4} = .75.$ Ans.

(*c*) $\dfrac{1.25 \times 20 \times 3}{\dfrac{87+88}{459+32}} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

$$\begin{array}{rr} & 1.25 \\ \times & 20 \\ \hline & 25.00 \\ \times & 3 \\ \hline & 75 \end{array}$$

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\frac{87+88}{459+32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87 + 88 = 175$.

The value of the denominator of this fraction is equal to $459 + 32 = 491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \frac{\frac{75}{1}}{\frac{175}{491}} = \frac{75}{1} \div \frac{175}{491} = \frac{75}{1} \times \frac{491}{175} = \frac{\overset{3}{\cancel{75}} \times 491}{\underset{7}{\cancel{175}}} = \frac{1{,}473}{7} = 210\tfrac{3}{7}. \text{ Ans.}$$

(59) The pitch of the rivets is the distance between the centers of the rivets. Hence, since the distance around the cylindrical boiler is 166.85 in., and there are 72 rivets in one of the seams, the pitch of the rivets equals $166.85 \div 72 = 2.317 +$ in. Ans.

$$\begin{array}{r l}
72\,)\,166.850\,(& 2.317+ \\
144 & \\
\hline
228 & \\
216 & \\
\hline
125 & \\
72 & \\
\hline
530 & \\
504 & \\
\hline
26 &
\end{array}$$

(60) If a keg containing 133 boiler rivets weighs 100 pounds, then each rivet must weigh as much as 133 is contained times in 100, or .75 of a pound.

$$\begin{array}{r l}
133\,)\,100.00\,(& .75+ \quad \text{Ans.} \\
931 & \\
\hline
690 & \\
665 & \\
\hline
25 &
\end{array}$$

Since there are 2 decimal places in the dividend and 0 decimal places in the divisor, we must point off $2 - 0 = 2$ decimal places in the quotient, or answer.

(61) If the distance around a fly-wheel is 56.5488 ft., which is 3.1416 times its diameter, then its diameter must equal 56.5488 ÷ 3.1416 = 18 ft.

3.1416) 56.5488 (18 ft.
31416
251328
251328
0

If the diameter of the fly-wheel is 18 ft., then it is evident that the diameter of a wheel one-half as large must be $\frac{1}{2}$ of 18 ft., or 9 ft.

(62) Since 1 load consists of 1,600 bricks, 8 loads consist of 8 × 1,600 = 12,800 bricks. Since 1 team can draw 12,800 bricks in 1 day, 6 teams can draw 6 times as many, or 6 × 12,800 = 76,800 bricks in 1 day. Since 6 teams can draw 76,800 bricks in 1 day, to draw 980,000 bricks, it will take them as many days as 76,800 is contained times in 980,000, or 12.76 + days. Ans.

76800) 980000.00 (12.76 +
76800
212000
153600
584000
537600
464000
460800
3200

Since there are 2 decimal places in the dividend and 0 decimal places in the divisor, we must point off 2 − 0 = 2 decimal places in the quotient.

(63) 28 acres of land at $133 an acre would cost 28 × $133 = $3,724.

28
1064
266
$3724

If a mechanic earns \$1,500 a year, and his expenses are \$968 per year, then he would save \$1500 — \$968, or \$532 per year.

$$\begin{array}{r} \$1500 \\ 968 \\ \hline \$532 \end{array}$$

If he saves \$532 in one year, to save \$3,724 it would take as many years as \$532 is contained times in \$3,724, or 7 years.

$$\begin{array}{r} 532\,)\,3724\,(\,7 \text{ years. Ans.} \\ 3724 \\ \hline 0 \end{array}$$

(64) $\frac{7}{8}$ = value of the fraction, and 28 the numerator. We find that 4 multiplied by 7 = 28, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32} = \frac{7}{8}$. Hence, 32 is the required denominator.

(65) 1 plus .001 = 1.001. .01 plus .000001 = .010001. And 1.001 — .010001 =

$$\begin{array}{r} 1.001 \\ .010001 \\ \hline .990999 \end{array} \text{ Ans.}$$

(66) If the freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles the next week, then it ran (3 × 365 miles) — 246 miles, or 849 miles the second week. Thus,

$$\begin{array}{r} 365 \\ 3 \\ \hline 1095 \\ -\ 246 \\ \hline 849 \end{array} \text{ miles. Ans.}$$

(67) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in 1 mile, in 354 miles there are 354 × 5,280 feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, in going

(71) 72.6 feet of fencing at $.50 a foot would cost

```
              72.6 × .50, or $36.30.
               .50
             -------
             $36.300
```

If, by selling a carload of coal at a profit of $1.65 per ton, I make $36.30, then there must be as many tons of coal in the car as 1.65 is contained times in 36.30, or 22 tons.

```
1.65)36.30(22 tons.  Ans.
     330
     ---
      330
      330
      ---
        0
```

(72) 14 ft. 5 in. $= 14\frac{5}{12}$ ft., since 12 inches equal 1 foot. Of the six pieces of connecting pipe between an engine and boiler, there are three pieces each measuring 14 ft. 5 in., or $14\frac{5}{12}$ ft.; hence, the total length of the three pieces equals $3 \times 14\frac{5}{12} = 3 \times \frac{173}{12} = \frac{519}{12} = 43\frac{1}{4}$ ft.

Since each of the other two pieces is 12 ft. 6 in., or $12\frac{1}{2}$ ft. in length, the total length of these two pieces equals $2 \times 12\frac{1}{2}$ ft. $= 25$ ft. The remaining piece of pipe is 8 ft. 10 in., or $8\frac{5}{6}$ ft. in length, since 10 in. $= \frac{10}{12}$, or $\frac{5}{6}$ of a ft. (there being 12 inches in 1 foot). Adding these different lengths will give the total length of the six pieces of pipe. However, before adding we will reduce the fractions of the two mixed numbers $43\frac{1}{4}$ and $8\frac{5}{6}$ to fractions having a common denominator. Doing this, we have $\frac{1}{4} = \frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$; $\frac{5}{6} = \frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$. $\frac{3}{12} + \frac{10}{12} = \frac{13}{12} = 1\frac{1}{12}$ ft. The sum of the whole numbers equals 43 ft. + 25 ft. + 8 ft. = 76 ft., which added to $1\frac{1}{12}$ ft. gives $77\frac{1}{12}$ ft. as the total length of the connecting pipe. Since 1 ft. of pipe weighs $10\frac{1}{2}$ pounds, then it is clearly seen that this pipe would weigh $77\frac{1}{12} \times 10\frac{1}{2} = \frac{925}{\underset{4}{\cancel{12}}} \times \frac{\overset{7}{\cancel{21}}}{2} = \frac{6,475}{8} = 809\frac{3}{8}$, or 809.375 lb. Ans.

$$\begin{array}{r} 77 \\ \times\ 12 \\ \hline 154 \\ 77 \\ \hline 924 \\ +\ \ \ 1 \\ \hline 925 \end{array}$$

$\frac{3}{8} = .375$, since $8\,\overline{)\,3.000}$
$.375$

(73) Since these four bolts measure $2\frac{1}{2}$, $6\frac{7}{8}$, $3\frac{1}{16}$, and 4 inches, respectively, together they will measure $16\frac{7}{16}$ inches, since $2\frac{1}{2} + 6\frac{7}{8} + 3\frac{1}{16} + 4 = 16\frac{7}{16}$. Reducing the fractions of the mixed numbers to a common denominator, we have $\frac{1}{2} = \frac{1}{2} \times \frac{8}{8} = \frac{8}{16}$; $\frac{7}{8} = \frac{7}{8} \times \frac{2}{2} = \frac{14}{16}$; $\frac{8}{16} + \frac{14}{16} + \frac{1}{16} = \frac{8 + 14 + 1}{16} = \frac{23}{16} = 1\frac{7}{16}$. $2 + 6 + 3 + 4 + 1\frac{7}{16} = 16\frac{7}{16}$. If $\frac{7}{16}$ of an inch is allowed for cutting and finishing each bolt, then the allowance for the 4 bolts would equal $4 \times \frac{7}{16} = \frac{28}{16} = 1\frac{12}{16}$ inches, which added to $16\frac{7}{16}$ inches equals $18\frac{3}{16}$ inches, the length of the piece of iron required. Ans.

$$\begin{array}{r} 16\frac{7}{16} \\ 1\frac{12}{16} \\ \hline 17\frac{19}{16} \end{array}$$

$17\frac{19}{16}$, or $18\frac{3}{16}$, since $\frac{19}{16} = 1\frac{3}{16}$.

(74) A double belt of a certain width can transmit $64\frac{1}{2}$, or 64.5 horsepower. If this is $\frac{10}{7}$ of the power transmitted by one single belt, then $\frac{1}{7}$ of the power transmitted would equal $\frac{1}{10}$ of 64.5, or 6.45 horsepower, and $\frac{7}{7}$ would equal 7×6.45 horsepower, or 45.15 horsepower. Since 1 single belt of a certain width can transmit 45.15 horsepower, it is clearly evident that 2 single belts, when running under the same conditions, would transmit 2×45.15, or 90.3 horsepower. Ans.

(75) Before solving, we will reduce the several lengths to the same denomination, in this case feet. Since 12 inches = 1 foot,

$$18 \text{ ft. } 6 \text{ in.} = 18\frac{6}{12}, \text{ or } 18\frac{1}{2} \text{ ft.};$$

$$16 \text{ ft. } 9\tfrac{1}{2} \text{ in.} = 16\frac{9\frac{1}{2}}{12} = 16\frac{\frac{19}{2}}{12} = 16\frac{19}{24};$$

$$\left(\frac{19}{2} \div 12 = \frac{19}{2} \div \frac{12}{1} = \frac{19}{2} \times \frac{1}{12} = \frac{19}{24}\right);$$

$$22 \text{ ft. } 2 \text{ in.} = 22\frac{2}{12}, \text{ or } 22\frac{1}{6} \text{ ft.};$$

$$20 \text{ ft. } 8\tfrac{1}{2} \text{ in.} = 20\frac{8\frac{1}{2}}{12} = 20\frac{\frac{17}{2}}{12} = 20\frac{17}{24}.$$

$$\left(\frac{17}{2} \div 12 = \frac{17}{2} \div \frac{12}{1} = \frac{17}{2} \times \frac{1}{12} = \frac{17}{24}\right).$$

Adding the different lengths, we have

$$18\frac{1}{2} \text{ ft.} + 16\frac{19}{24} \text{ ft.} + 22\frac{1}{6} \text{ ft.} + 20\frac{17}{24} \text{ ft.} = 78\frac{1}{6} \text{ ft.}$$

Reducing $\frac{1}{2}$, $\frac{19}{24}$, $\frac{1}{6}$, and $\frac{17}{24}$ to fractions having a common denominator, as in problem 42, we have $\frac{1}{2} = \frac{12}{24}$; $\frac{19}{24} = \frac{19}{24}$; $\frac{1}{6} = \frac{4}{24}$, and $\frac{17}{24} = \frac{17}{24}$. Adding, we have $\frac{12 + 19 + 4 + 17}{24} =$

$\frac{52}{24} = 2\frac{4}{24}$, or $2\frac{1}{6}$, which added to the sum of the whole numbers, or 76, equals $78\frac{1}{6}$ ft. Ans.

$$\begin{array}{rl}
18\frac{1}{2} & \text{ft.} \\
16\frac{19}{24} & \text{ft.} \\
22\frac{1}{6} & \text{ft.} \\
20\frac{17}{24} & \text{ft.} \\
\hline
76\phantom{\frac{1}{6}} & \text{ft.} \\
2\frac{1}{6} & \text{ft.} \\
\hline
78\frac{1}{6} & \text{ft.}
\end{array}$$

(76) The total horsepower developed equals 48.63 + 45.7 + 46.32 + 47.9 + 48.74 + 48.38 + 48.59 = 334.26.

Since the horsepower developed equals 334.26, then the average horsepower developed must equal 334.26 ÷ 7, or 47.75 + H. P.

$$\begin{array}{l}
48.63 \\
45.7 \\
46.32 \\
47.9 \\
48.74 \\
48.38 \\
48.59 \\
\hline
334.26
\end{array}$$

(81) Since percentage = base × rate, rate = percentage ÷ base.*

As percentage = 10 and base = 10, we have rate = 10 ÷ 10 = 1.

But $1 = \frac{100}{100}$ and $\frac{100}{100} = 100\%$; hence, the rate (1) means that 10 is 100% of 10.

(82) (*a*) Rate = percentage ÷ base.

As percentage = $176.54 and base = $2,522, we have rate = 176.54 ÷ 2,522 = .07 = 7%. Ans.

```
2522 ) 176.54 ( .07
       17654
       -----
```

(*b*) Base = percentage ÷ rate.

As percentage = 16.96 and rate = 8% = .08, we have base = 16.96 ÷ .08 = 212. Ans.

```
.08 ) 16.96 ( 212
      16
      --
        9
        8
        --
        16
        16
        --
```

(*c*) Amount is the sum of the base and percentage; hence, the percentage = amount minus the base.

Amount = 216.7025 and base = 213.5; hence, percentage = 216.7025 − 213.5 = 3.2025.

Rate = percentage ÷ base.

As percentage = 3.2025 and base = 213.5, we have rate = 3.2025 ÷ 213.5 = .015. Ans.

```
213.5 ) 3.2025 ( .015 = 1½%
        2135
        -----
        10675
        10675
        -----
```

**Remember that an expression of this form means that the first term is to be divided by the second term. Thus, as above, it means percentage divided by base.*

(*d*) The difference is the remainder found by subtracting the percentage from the base; hence, base − the difference = the percentage. Base = 207 and difference = 201.825; hence, percentage = 207 − 201.825 = 5.175.

Rate = percentage ÷ base.

As percentage = 5.175 and base = 207, we have rate = 5.175 ÷ 207 = $.025 = .02\frac{1}{2} = 2\frac{1}{2}\%$. Ans.

```
207 ) 5.175 ( .025
      414
      ----
      1035
      1035
      ----
```

(**83**) Since 5,500 lb. represent an increase of 15% over the consumption when the condenser is used, 5,500 lb. must be the amount, 15% the rate, and the number of pounds consumed when the condenser is running (to be found) the base.

Base = amount ÷ (1 + rate) = 5,500 ÷ (1 + .15)
= 5,500 ÷ 1.15 = 4,782.61 lb., nearly. Ans.

```
1.15 ) 5500.0000 ( 4782.61
       460
       ---
        900
        805
        ---
         950
         920
         ---
          300
          230
          ---
           700
           690
           ---
            100
            115
            ---
```

Or, this problem could also have been solved as follows:

100% = the number of pounds consumed when the condenser is running. If there is a gain of 15%, then 100% + 15%, or 115% = 5,500 lb., the amount used when the condenser is not running. If 115% = 5,500 lb., 1% = $\frac{1}{115}$ of 5,500 = 47.8261 lb., and 100% = 100 × 47.8261 = 4,782.61 lb. Ans.

(84)

$$
\begin{array}{ll}
24\ \% \text{ of } \$950 = 950 \times .24 & = \$228 \\
12\tfrac{1}{2}\% \text{ of } \$950 = 950 \times .125 = & 118.75 \\
17\ \% \text{ of } \$950 = 950 \times .17 & = \underline{161.50} \\
\overline{53\tfrac{1}{2}}\% \text{ of } \$950 & = \$508.25
\end{array}
$$

The total amount of his yearly expenses, then, is $508.25; hence, his savings are $950 – $508.25 = $441.75. Ans.

Or, as above, $24\% + 12\frac{1}{2}\% + 17\% = 53\frac{1}{2}\%$, the total percentage of expenditures; hence,

$950 × .535 = $508.25, and
$950 – $508.25 = $441.75 = his yearly savings. Ans.

(85) The percentage is 961.38 and the rate is $37\frac{1}{2}\%$.

Base = percentage ÷ rate
= 961.38 ÷ .375 = 2,563.68, the number. Ans.

```
.375 ) 961.38000 ( 2563.68
       750
       ----
       2113
       1875
       ----
        2388
        2250
        ----
         1380
         1125
         ----
          2550
          2250
          ----
           3000
           3000
           ----
```

Another method of solving is the following: If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38 and the number = $961.38 \div .37\frac{1}{2}$, which, as above = 2,563.68. Ans.

(86) Here $1,125 is 30% of some number; hence, $1,125 = the percentage, 30% = the rate, and the required number is the base.

Base = percentage ÷ rate.
= $1,125 ÷ .30 = $3,750.

Since $3,750 is $\frac{3}{4}$ of the property, one of the fourths is $\frac{1}{3}$

of $3,750 = $1,250, and $\frac{4}{4}$, or the entire property, is 4 × $1,250 = $5,000. Ans.

(87) Here, $4,810 is the difference and 35% the rate.
Base = difference ÷ (1 − rate)
= $4,810 ÷ (1 − .35) = $4,810 ÷ .65 = $7,400. Ans.

```
.65 ) 4810.00 ( 7400
      455
      ---
       260                    1.00
       260                     .35
       ---                    ----
                               .65
```

The solution can also be effected as follows: 100% = the sum diminished by 35%, then (1 − .35) = .65, which is $4,810. If 65% = $4,810, 1% = $\frac{1}{65}$ of 4,810 = $74, and 100% = 100 × $74 = $7,400. Ans.

(88) The volume of the clearance in a steam-engine cylinder = 18.3 cu. in., and the volume of the cylinder, neglecting the clearance, = 254.5 cu. in. We wish to know what percentage of the cylinder volume is the clearance, or what per cent. of 254.5 is 18.3.

254.5 is the base.

18.3 is the percentage.

The rate, which we wish to find, equals the percentage ÷ base = 18.3 ÷ 254.5 = 7.2% nearly.

```
254.5 ) 18.3000 (.072, or 7.2%, nearly.
        17815
        -----
          4850
          5090
          ----
```

(89) 16.5 miles = $12\frac{1}{2}$% of the entire length of the road We wish to find the *entire* length.

16.5 miles is the percentage and $12\frac{1}{2}$% is the rate, and the entire length will be the base.

$$\begin{aligned}\text{Base} &= \text{percentage} \div \text{rate}\\ &= 16.5 \div .12\tfrac{1}{2}.\end{aligned}$$

$$\begin{array}{r l}
.125)\,16.500\,(& 132 \text{ miles.} \quad \text{Ans.}\\
\underline{125} & \\
400 & \\
\underline{375} & \\
250 & \\
\underline{250} &
\end{array}$$

(90) 298 revolutions per minute with the load = base. $1\frac{1}{2}\%$ = rate, and the amount (to be found) will equal the speed of the engine when running unloaded.

$$\begin{aligned}\text{Amount} &= \text{base} \times (1 + \text{rate})\\ &= 298 \times (1 + .015) = 302.47 \text{ rev. per min.} \quad \text{Ans.}\end{aligned}$$

$$\begin{array}{r}
298\\
\times\ \underline{1.015}\\
1490\\
298\\
000\\
\underline{298}\\
302.470
\end{array}$$

(91) 4 yd. 2 ft. 10 in. to inches.

$$\begin{array}{r l}
4 & \text{yd.}\\
\times\ \underline{3} & \\
12 & \\
+\ \underline{2} & \\
14 & \text{feet}\\
\times\ \underline{12} & \\
28 & \\
\underline{14} & \\
168 & \\
+\ \underline{10} & \\
178 & \text{inches.} \quad \text{Ans.}
\end{array}$$

Since there are 3 feet in one yard, in 4 yards there are 4 × 3 feet, or 12 feet. 12 feet plus 2 feet = 14 feet.

There are 12 inches in one foot; therefore, in 14 feet there are 12 × 14 or 168 inches. 168 inches plus 10 inches = 178 inches. Ans.

(92)

$$\begin{array}{r l}
12\,\underline{)\,3722} & \text{inches.} \\
3\,\underline{)\,310} & +\,2 \text{ inches.} \\
103 & +\,1 \text{ foot.}
\end{array}$$

Ans. = 103 yd. 1 ft. 2 in.

Explanation.—There are 12 inches in one foot; hence, in 3,722 inches there are as many feet as 12 is contained times in 3,722, or 310 feet and 2 inches remaining. Write 2 inches as a remainder. There are 3 feet in one yard; hence, in 310 yards there are as many feet as 3 is contained times in 310, or 103 yards and 1 foot remaining. Hence, in 3,722 inches there are 103 yd. 1 ft. 2 in.

(93)

$$\begin{array}{r l}
 & 5 \text{ weeks } 3.5 \text{ days.} \\
\times & \underline{7} \\
 & 35 \text{ days in 5 weeks.} \\
+ & \underline{3.5} \\
 & 38.5 \text{ days.}
\end{array}$$

Then, we find how many seconds there are in 38.5 days.

$$\begin{array}{r l}
38.5 & \text{days.} \\
\underline{\times\ \ 24} & \text{hours in one day.} \\
1540 & \\
\underline{770\ } & \\
924.0 & \text{hours in 38.5 days.} \\
\underline{\times\ \ 60} & \text{minutes in one hour.} \\
55440 & \text{minutes in 38.5 days.} \\
\underline{\times\ \ 60} & \text{seconds in one minute.} \\
3326400 & \text{seconds in 38.5 days.}\quad \text{Ans.}
\end{array}$$

(94)

$$\begin{array}{r l}
1728\,\underline{)\,764325} & \text{cu. in.} \\
27\,\underline{)\,442} & +\,549 \text{ cu. in.} \\
16 & \text{cu. yd.} +\,10 \text{ cu. ft.}
\end{array}$$

Ans. = 16 cu. yd. 10 cu. ft. 549 cu. in.

Explanation.—There are 1,728 cubic inches in one cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cubic feet, and 549 cubic inches remaining. Write the 549 cubic inches as a remainder. There are 27 cubic feet in one cubic yard; hence, in 442 cubic feet there are as many cubic yards as 27 is contained times in 442 cubic feet, or 16 cubic yards and 10 cubic feet remaining. Then, in 764,325 cubic inches there are 16 cu. yd. 10 cu. ft. 549 cu. in. Ans.

(95) There are $31\frac{1}{2}$ (31.5, since $\frac{1}{2} = .5$) gallons in 1 barrel; hence, in 4 barrels there are 4 times 31.5, or 126 gallons. 126 gallons plus 10 gallons = 136 gallons.

There are 4 quarts in 1 gallon; hence, in 136 gallons there are 136 times 4 = 544 quarts.

544 quarts plus 3 quarts = 547 quarts. Since the tank under consideration contains 547 quarts, and since 4 quarts = 1 gallon, the tank would contain as many gallons of water as 4 is contained times in 547, or $136\frac{3}{4}$ gallons. Ans.

$$\begin{array}{rrrl}
\text{bbl.} & \text{gal.} & \text{qt.} & \\
4 & 10 & 3 & \\
\times\ 31.5 & & & \\ \hline
126.0 & & & \\
+\ 10 & & & \\ \hline
136 & \text{gallons.} & & \\
\times\ 4 & & & \\ \hline
544 & & & \\
+\ 3 & & & \\ \hline
4)\overline{547} & \text{quarts.} & & \\ \hline
136\frac{3}{4} & \text{gallons.} & & \\
 & \text{Ans.} & &
\end{array}$$

(96) Since in one ton there are 20 cwt., in 16 tons there are 16 × 20 = 320 cwt. 320 cwt. + 8 cwt. = 328 cwt. There are 100 lb. in 1 cwt.; hence, in 328 cwt. there are 328 × 100 = 32,800 lb. 32,800 lb. + 75 lb. = 32,875 lb. Ans.

$$\begin{array}{rrrr}
 & \text{T.} & \text{cwt.} & \text{lb.} \\
 & 16 & 8 & 75 \\
\times & 20 & & \\ \hline
 & 320 & & \\
+ & 8 & & \\ \hline
 & 328 & \text{cwt.} & \\
\times & 100 & & \\ \hline
 & 32800 & & \\
+ & 75 & & \\ \hline
 & 32875 & \text{lb.} &
\end{array}$$

(97)

$$\begin{array}{r}
100)\underline{25396}\ \text{lb.} \\
20)\underline{253}\ \text{cwt.} + 96\ \text{lb.} \\
12\ \text{T.} + 13\ \text{cwt.}
\end{array}$$

There are 100 lb. in 1 cwt.; hence, in 25,396 lb. there are as many cwt. as 100 is contained times in 25,396, or 253 cwt. and 96 lb. remaining.

There are 20 cwt. in 1 ton, and in 253 cwt. there are as many tons as 20 is contained times in 253, or 12 tons and

13 cwt. remaining. Hence, 25,396 lb. = 12 T. 13 cwt. 96 lb. Ans.

(98) There are 2 pt. in 1 qt.; hence, in 25,396 pt. there are 12,698 qt. There are 4 qt. in one gal.; hence, in 12,698 qt. there are as many gal. as 4 is contained times in 12,698, or 3,174 gal. with 2 qt. remaining.

2) 2 5 3 9 6 pt.
4) 1 2 6 9 8 qt.
3 1.5) 3 1 7 4 gal. + 2 qt.
1 0 0 bbl. + 24 gal.

Since $31\frac{1}{2}$, or 31.5 gal. make 1 bbl., in 3,174 gal. there would be as many bbl. as 31.5 is contained times in 3,174, or 100 bbl. and 24 gal. remaining.

3 1.5) 3 1 7 4.0 gal. (1 0 0 bbl.
3 1 5
2 4.0

Hence, 25,396 pt. = 100 bbl. 24 gal. 2 qt. Ans.

(99) Arrange the different terms in columns, taking care to have like denominations in the same column. We begin to add at the right-hand column. 7 + 9 + 3 = 19 in.; since 12 in. = 1 ft., 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column, and reserve the 1 ft. to add to the sum of the feet. 2 + 1 + 2 + 1 (reserved) = 6 ft. Since 3 ft. = 1 yd., 6 ft. = 2 yd. and 0 ft. remaining. Place the 0 in the feet column and reserve the 2 yd. to add to the sum of the yards. 4 + 2 + 2 (reserved) = 8 yd., which we place in yards column. Ans. = 8 yd. 7 in.

yd.	ft.	in.	
2	2	3	
4	1	9	
	2	7	
8	0	7	Ans.

(100) Since "pints" is the lowest denomination, we will find their sum first. 5 + 1 + 1 = 7 pt. Since there are 2 pt. in 1 qt., in 7 pt. there are as many quarts as 2 is contained in 7, or 3 qt. and 1 pt. remaining. Place 1 pt. under pints column and add

gal.	qt.	pt.	
3	3	1	
6	0	1	
4	8	5	
16	2	1	Ans.

3 qt. to the sum of the quarts. 8 + 3 + 3 (reserved) = 14 qt. Since there are 4 qt. in 1 gal., in 14 qt. there are as many gallons as 4 is contained in 14, or 3 gal., and 2 qt. remaining. Place 2 qt. under quarts column and reserve the 3 gal. to add to the sum of the gallons. 4 + 6 + 3 + 3 (reserved) = 16 gal. Hence, the answer is 16 gal. 2 qt. 1 pt.

(101) We will first reduce 115 ft. to yards. Since 3 ft. = 1 yd., 115 ft. = 38 yd. and 1 ft. Place the subtrahend under the minuend, so that like denominations are under each other. Since the inch is the lowest denomination, we subtract the inches first. 0 in. subtracted from 9 in. = 9 in., which we write in inches place in the remainder. 1 ft. subtracted from 2 ft. = 1 ft., which we write in the remainder. 38 yd. subtracted from 52 yd. = 14 yd., which we write in yards place in the remainder. Hence, the remainder, or answer = 14 yd. 1 ft. 9 in.

yd.	ft.	in.	
52	2	9	
38	1	0	
14	1	9	Ans.

(102) Since 10 gal. 2 qt. 1 pt. of machine oil is sold at one time, and 16 gal. 3 qt. at another time, together there was sold 27 gal. 1 qt. 1 pt. 0 + 1 = 1 pt. We can not reduce 1 pt. to any higher denomination, so place it under pints column. 3 qt. + 2 qt. = 5 qt. Since 4 qt. = 1 gal., 5 qt. = 1 gal., and 1 qt. remaining. Place 1 qt. under quarts column and reserve the 1 gal. to add to the gallons. 16 gal. + 10 gal. + 1 gal. (reserved) = 27 gal. Since the barrel contained 31½, or 31.5 gal., and 27 gal. 1 qt. 1 pt. were sold, there remained the difference, or 4 gal. 1 pt. 31.5 gal. = 31 gal. 2 qt., since .5 = $\frac{1}{2}$, and $\frac{1}{2}$ of 1 gal. = $\frac{1}{2}$ of 4 qt. = 2 qt. 1 pt. can not be taken from 0 pt., so we take 1 qt. from the 2 qt. The 1 qt. taken = 2 pt. 1 pt. from 2

gal.	qt.	pt.
10	2	1
16	3	0
27	1	1

gal.	qt.	pt.	
31	2	0	
27	1	1	
4	0	1	Ans.

pt. = 1 pt. Place 1 pt. under pints column. Since we took 1 qt. from the quarts column, there remains 2 − 1, or 1 qt. 1 qt. from 1 qt. leaves 0 qt. Place 0 qt. under the quarts column. 27 gal. from 31 gal. leaves 4 gal. Place 4 gal. under the gallons column. We, therefore, find that 4 gal. 1 pt. of machine oil remained in the barrel.

(103) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

$$\begin{array}{r} 17\text{ ft. } 3\text{ in.} \\ 51\phantom{\text{ in.}} \\ \hline 879\text{ ft. } 9\text{ in.} \end{array}$$

and begin at the right to multiply. 51 × 3 = 153 in. Since there are 12 in. in 1 ft., in 153 in. there are as many feet as 12 is contained times in 153, or 12 ft. and 9 in. remaining. Place the 9 in. under the inches and reserve the 12 ft. 51 × 17 ft. = 867 ft.; 867 ft. + 12 ft. (reserved) = 879 ft. 879 ft. can be reduced to higher denominations by dividing by 3 ft. to find the number of yards, and by $5\frac{1}{2}$ to find the number of rods.

$$\begin{array}{r} 3\,\underline{)\,879}\text{ ft. } 9\text{ in.} \\ 5.5\,\underline{)\,293}\text{ yd.}\phantom{\text{ 9 in.}} \\ 53\text{ rd. } 1\tfrac{1}{2}\text{ yd.} \end{array}$$

Then, Ans. = 53 rd. 1½ yd. 0 ft. 9 in., or 53 rd. 1 yd. 2 ft. 3 in.

(104) Since 2 pt. = 1 qt., 3 qt. = 3 × 2, or 6 pt. 6 pt. + 1 pt. = 7 pt. 4.7 × 7 = 32.9 pt. Ans.

$$\begin{array}{rl} \text{qt.} & \text{pt.} \\ 3 & 1 \\ \times\ 2 & \\ \hline 6 & \\ +\ 1 & \\ \hline 7 & \text{pt.} \end{array} \qquad \begin{array}{r} 7\text{ pt.} \\ 4.7\phantom{\text{ pt.}} \\ \hline 49\phantom{\text{ pt.}} \\ 28\phantom{0\text{ pt.}} \\ \hline 32.9\text{ pt.} \end{array}$$

(105) We must first reduce 23 miles to feet before we can divide by 30 feet.

$$1 \text{ mi.} = 320 \text{ rd.}$$

$$23 \text{ mi.} = 23 \times 320 = 7{,}360 \text{ rd.}$$

$$1 \text{ rd.} = 16\frac{1}{2} \text{ or } \frac{33}{2} \text{ ft.}$$

$$7{,}360 \text{ rd.} = \overset{3680}{\cancel{7360}} \times \frac{33}{\underset{1}{\cancel{2}}} = 121{,}440 \text{ ft.}$$

121,440 ft. ÷ 30 = 4,048 rails for one side of the track.

The number of rails for 2 sides of the track = 2 × 4,048, or 8,096 rails. Ans.

(106) If there are four lengths, each 15 ft. 5 in., 15 ft. 5 in. × 4 = 60 ft. 20 in., or the length of the four pieces.

$$\begin{array}{rr} \text{ft.} & \text{in.} \\ 60 & 20 \\ 14 & 8 \\ 8 & 10 \\ \hline 82 & 38 \end{array} \qquad \begin{array}{rr} 15 \text{ ft.} & 5 \text{ in.} \\ & 4 \\ \hline 60 \text{ ft.} & 20 \text{ in.} \end{array}$$

82 38, or 85 ft. 2 in. = the length of the shaft.

Since there are six hangers, there are *five* spaces. We will now find the distance between the hangers, or the length of one space. This we find by dividing the total length by 5.

$$\begin{array}{r} 5\,\underline{)\,85 \text{ ft. } 2 \text{ in.}} \\ 17 \text{ ft. } \frac{2}{5} \text{ in.} \end{array} \text{ Ans.}$$

Hence, 17 ft. $\frac{2}{5}$ in. is the distance between the hangers.

(107) Since the distance around a wheel is $\frac{22}{7}$ times the distance across it, the distance around the wheel will equal (9 ft. $6\frac{1}{2}$ in.) × $\frac{22}{7}$. We will reduce 9 ft. to inches before multiplying by the fraction. 9 ft. × 12 = 108 in. 108 in. + $6\frac{1}{2}$ in. = $114\frac{1}{2}$ in.

Reducing $114\frac{1}{2}$ to an improper fraction, we have $\frac{229}{2}$.

Multiplying, $\frac{229}{\cancel{2}} \times \frac{\overset{11}{\cancel{22}}}{7} = \frac{2{,}519}{7} = 359\frac{6}{7}$ in.

$$\begin{array}{r} 229 \\ \times\ 11 \\ \hline 229 \\ 229 \\ \hline 2519 \end{array}$$

Dividing $359\frac{6}{7}$ in. by 12 to reduce to feet, we have 29 ft. $11\frac{6}{7}$ in.

$$\begin{array}{l} 12)\,359\frac{6}{7} \text{ in. } (29 \text{ ft.} \\ \underline{24} \\ 119 \\ \underline{108} \\ 11\frac{6}{7} \text{ in.} \end{array}$$

(108) Reducing 18 ft. $11\frac{1}{4}$ in. to inches, we have $227\frac{1}{4}$ in., or 227.25 in.

$$\begin{array}{rl} \text{ft.} & \text{in.} \\ 18 & 11\frac{1}{4} \\ \underline{12} & \\ 36 & \\ \underline{18} & \\ 216 & \\ \underline{11\frac{1}{4}} & \\ 227\frac{1}{4} \text{ in.} & \end{array}$$

$1\frac{1}{8} \times 2$, or $2\frac{1}{4}$ in., for the two end rivets is deducted from the length, leaving 225 in., which is divided into equal spaces by the rivets.

$$\begin{array}{r} 227.25 \text{ in.} \\ -\ \ 2.25 \text{ in.} \\ \hline 225 \text{ in.} \end{array}$$

The pitch of the rivets (or the distance between their centers) is $1\frac{1}{4}$ in., or 1.25 in.; hence, 225 ÷ 1.25 = 180 spaces between the rivets. But, since there will be 1 more rivet than the number of spaces, the number of rivets required for this boiler shell will be 180 + 1 = 181. Ans.

```
1.25)225.00(180
     125
     ----
     1000
     1000
     ----
        0
```

(109) (*a*) To find the second power of a number, we must multiply the number by itself once; that is, use the number twice as a factor. Thus, the second power of 108 is 108 × 108 = 11,664. Ans.

```
  108
  108
 ----
  864
108
-----
11664  Ans.
```

(*b*) The third power of 181.25 equals the number obtained by using 181.25 as a factor three times. Thus, the third power of 181.25 is 181.25 × 181.25 × 181.25 = 5,954,345.703125. Ans.

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are 2 + 2 = 4 decimal places in the first product.

Since there are 4 decimal places in the multiplicand, and 2 in the multiplier, there are 4 + 2 = 6 decimal places in the final product.

```
          181.25
          181.25
          ------
           90625
          36250
         18125
       145000
       18125
      ----------
       32851.5625
          181.25
      ----------
      1642578125
     6570312 50
    3285156 25
  2628125000
  3285156 25
  --------------
  5954345.703125 Ans.
```

$$\begin{array}{r}
27.61 \\
\underline{27.61} \\
2761 \\
16566 \\
19327 \\
\underline{5522} \\
762.3121 \\
\underline{27.61} \\
7623121 \\
45738726 \\
53361847 \\
\underline{15246242} \\
21047.437081 \\
\underline{27.61} \\
21047437081 \\
126284622486 \\
147332059567 \\
\underline{42094874162} \\
581119.73780641
\end{array}$$

Ans.

(*c*) The fourth power of 27.61 is the number obtained by using 27.61 as a factor four times. Thus, the fourth power of 27.61 is 27.61 × 27.61 × 27.61 × 27.61 = 581,119.73780641. Ans.

Since there are 2 decimal places in the multiplier, and 2 in the multiplicand, there are 2 + 2 = 4 decimal places in the first product.

Since there are 4 decimal places in the multiplicand, and 2 in the multiplier, there are 4 + 2 = 6 decimal places in the second product.

Since there are 6 decimal places in the multiplicand, and 2 in the multiplier, there are 6 + 2 = 8 decimal places in the final product.

(110) (*a*) $106^2 = 106 \times 106 = 11{,}236$. Ans.

$$\begin{array}{r}
106 \\
\underline{106} \\
636 \\
\underline{106} \\
11236
\end{array}$$

Ans.

(*b*) $\left(182\frac{1}{8}\right)^2 = 182\frac{1}{8} \times 182\frac{1}{8} = 33{,}169.515625$. Ans.

$$\frac{1}{8} = 8\ )\ 1.000\ (\ .125$$

$$\begin{array}{r}
\underline{8} \\
20 \\
\underline{16} \\
40 \\
\underline{40}
\end{array}$$

$$\begin{array}{r}
182.125 \\
\underline{182.125} \\
910625 \\
364250 \\
182125 \\
364250 \\
1457000 \\
\underline{182125} \\
33169.515625
\end{array}$$

Ans.

Since there are 3 decimal places in the multiplier and 3 in the multiplicand, there are 3 + 3 = 6 decimal places in the product.

(*c*) $.005^3 = .005 \times .005 \times .005 = .000000125$. Ans.

$$\begin{array}{r}
.005 \\
.005 \\
\hline
.000025 \\
.005 \\
\hline
.000000125
\end{array}\ \text{Ans.}$$

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3 + 3 = 6$ decimal places in the first product; but, as there are only 2 figures in the product, we prefix four ciphers to make the necessary 6 decimal places.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, there are $6 + 3 = 9$ decimal places in the final product. In this case we prefix six ciphers to form the necessary 9 decimal places.

(*d*) $.4044^3 = .4044 \times .4044 \times .4044 = .066135317184$. Ans.

$$\begin{array}{r}
.4044 \\
.4044 \\
\hline
16176 \\
16176 \\
16176 \\
\hline
.16353936 \\
.4044 \\
\hline
65415744 \\
65415744 \\
65415744 \\
\hline
.066135317184
\end{array}\ \text{Ans.}$$

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4 + 4 = 8$ decimal places in the first product.

Since there are 8 decimal places in the second multiplicand and 4 in the multiplier, there are $8 + 4 = 12$ decimal places in the final product; but, as there are only 11 figures in the product, we prefix 1 cipher to make the necessary 12 decimal places.

(*e*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637$. Ans.

$$\begin{array}{r}
.0133 \\
\underline{.0133} \\
399 \\
399 \\
133 \\
\hline
.00017689 \\
\underline{.0133} \\
53067 \\
53067 \\
17689 \\
\hline
.000002352637
\end{array} \text{ Ans.}$$

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we should point off $4 + 4 = 8$ decimal places in the product; but, as there are only 5 figures in the product, we prefix three ciphers to form the eight necessary decimal places in the first product.

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we should point off $8 + 4 = 12$ decimal places in the product; but, as there are only 7 figures in the product, we prefix 5 ciphers to make the 12 necessary decimal places in the final product.

(*f*) $301.011^3 = 301.011 \times 301.011 \times 301.011 =$ $27{,}273{,}890.942264331$. Ans.

$$\begin{array}{r}
301.011 \\
\underline{301.011} \\
301011 \\
301011 \\
301011 \\
903033 \\
\hline
90607.622121 \\
301.011 \\
\hline
90607622121 \\
90607622121 \\
90607622121 \\
271822866363 \\
\hline
27273890.942264331
\end{array} \text{ Ans.}$$

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, we should point off $3 + 3 = 6$ decimal places in the first product.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, we should point off $6 + 3 = 9$ decimal places in the final product.

(*g*) $\left(\frac{1}{8}\right)^3 = \frac{1}{8}\times\frac{1}{8}\times\frac{1}{8} = \frac{1\times1\times1}{8\times8\times8} = \frac{1}{512}.$ Ans.

$$\begin{array}{r} 8 \\ \times\ \underline{8} \\ 64 \\ \times\ \underline{8} \\ 512 \end{array}$$

(*h*) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer and multiply the denominators together for the denominator of the answer.

$$\left(3\frac{3}{4}\right)^3 = \frac{15}{4}\times\frac{15}{4}\times\frac{15}{4} = \frac{15\times15\times15}{4\times4\times4} = \frac{3{,}375}{64} = 52.734375.$$

Ans.

$$3\frac{3}{4} = \frac{(3\times4)+3}{4} = \frac{12+3}{4} = \frac{15}{4}.$$

$$\begin{array}{r} 15 \\ \underline{15} \\ 75 \\ \underline{15} \\ 225 \\ \underline{15} \\ 1125 \\ \underline{225} \\ 3375 \end{array} \qquad \begin{array}{l} 64\,)\,3375.000000\,(\,52.734375 \\ \underline{320} \\ 175 \\ \underline{128} \\ 470 \\ \underline{448} \\ 220 \\ \underline{192} \\ 280 \\ \underline{256} \\ 240 \\ \underline{192} \\ 480 \\ \underline{448} \\ 320 \\ \underline{320} \end{array}$$

Since six ciphers were annexed to the dividend, six decimal places must be pointed off in the quotient.

(112) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. Ans.

$$\begin{array}{rl} 2 & \\ 2 & \\ \hline 4 & \\ 2 & \\ \hline 8 & \\ 2 & \\ \hline 16 & \\ 2 & \\ \hline 32 & \text{Ans.} \end{array}$$

(113) $3^4 = 3 \times 3 \times 3 \times 3 = 81$. Ans.

$$\begin{array}{rl} 3 & \\ 3 & \\ \hline 9 & \\ 3 & \\ \hline 27 & \\ 3 & \\ \hline 81 & \text{Ans.} \end{array}$$

(114) $7^9 = 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 =$ 40,353,607. Ans.

$$\begin{array}{rl} 7 & \\ 7 & \\ \hline 49 & \\ 7 & \\ \hline 343 & \\ 7 & \\ \hline 2401 & \\ 7 & \\ \hline 16807 & \\ 7 & \\ \hline 117649 & \\ 7 & \\ \hline 823543 & \\ 7 & \\ \hline 5764801 & \\ 7 & \\ \hline 40353607 & \text{Ans.} \end{array}$$

(115) (*a*) $1.2^4 = 1.2 \times 1.2 \times 1.2 \times 1.2 = 2.0736$. Ans.

$$\begin{array}{r}
1.2 \\
1.2 \\ \hline
24 \\
12 \\ \hline
1.44 \\
1.2 \\ \hline
288 \\
144 \\ \hline
1.728 \\
1.2 \\ \hline
3456 \\
1728 \\ \hline
2.0736
\end{array}$$

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we should point off $1 + 1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we should point off $2 + 1 = 3$ decimal places in the second product.

Since there are 3 decimal places in the multiplicand and 1 in the multiplier, we should point off $3 + 1 = 4$ decimal places in the final product.

(*b*) $11^5 = 11 \times 11 \times 11 \times 11 \times 11 = 161{,}051$. Ans.

$$\begin{array}{r}
11 \\
11 \\ \hline
11 \\
11 \\ \hline
121 \\
11 \\ \hline
121 \\
121 \\ \hline
1331 \\
11 \\ \hline
1331 \\
1331 \\ \hline
14641 \\
11 \\ \hline
14641 \\
14641 \\ \hline
161051
\end{array}$$

(*c*) $1^6 = 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$. Ans.

(*d*) $.01^4 = .01 \times .01 \times .01 \times .01 = .00000001$. Ans.

$$\begin{array}{r} .01\\ .01\\ \hline .0001\\ .01\\ \hline .000001\\ .01\\ \hline .00000001 \end{array}$$

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, we should point off 2 + 2 = 4 decimal places in the first product; but, as there is only 1 figure in the product, we prefix 3 ciphers to make the necessary 4 decimal places.

Since there are 4 decimal places in the second multiplicand and 2 in the multiplier, we should point off 4 + 2 = 6 decimal places in the second product. It is necessary to prefix 5 ciphers to make six decimal places.

Since there are 6 decimal places in the third multiplicand and 2 in the multiplier, we should point off 6 + 2 = 8 decimal places in the product. It is necessary to prefix 7 ciphers to make 8 decimal places in the final product.

(*e*) $.1^5 = .1 \times .1 \times .1 \times .1 \times .1 = .00001$. Ans.

$$\begin{array}{r} .1\\ .1\\ \hline .01\\ .1\\ \hline .001\\ .1\\ \hline .0001\\ .1\\ \hline .00001 \end{array}$$

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we should point off 1 + 1 = 2 decimal places in the first product. It is necessary to prefix 1 cipher to the product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we should point off 2 + 1 = 3 decimal places in the second product. It is necessary to prefix 2 ciphers to the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we should point off 3 + 1 = 4 decimal places in the third product. We prefix three ciphers to this product.

Since there are 4 decimal places in the fourth multiplicand and 1 in the multiplier, we should point off 4 + 1 or 5 decimal places in the final product. It is necessary to prefix 4 ciphers to this product.

(116) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(117) (*a*)

```
(a) 1          √3'48'67'84.40'10 = 1867.29+  Ans.
    1          (b) 1
   ---             ---
(d) 20         (c) 248
     8             224
   ---             ---
    28         (e)  2467
     8              2196
   ----             -----
    360              27184
      6              26089
   ----              ------
    366               109540
      6                74684
   -----              -------
    3720               3485610
       7               3361041
   -----               -------
    3727                124569
       7
   ------
    37340
        2
   ------
    37342
        2
   -------
    373440
         9
   -------
    373449
```

Explanation.—(1) Divide the power into periods. In the above case, where the power consists of a whole number and a decimal, we begin at the decimal point and proceed to the *left* in pointing off the *whole* number, and towards the *right* in pointing off the *decimal*, annexing a 0 to complete the last decimal period. In *square root two* figures constitute a period. Find the greatest single number whose square is *less* than or *equal* to 3, the first period. This is evidently 1, since $2^2 = 4$, which is greater than 3. Write 1 as the first figure of the root; also, write it to the left, as shown at (*a*). Now, multiply the 1 at (*a*) by the 1 in the root, and write the result under the first period, or 3, as shown at (*b*). Subtract, and bring down the next period 48, and annex it to the remainder 2, thus making 248 the *dividend*, as shown at (*c*). Add the root already found to the 1 at (*a*), thereby obtaining 2, and annex a cipher to this 2, thus making it 20, which we call the *trial divisor.*

Divide the dividend 248 at (*c*) by the trial divisor 20 at (*d*) and obtain 8, which is probably the next figure of the root. Write 8 in the root, as shown, and also add it to 20, the trial divisor, making it 28. This is called the *complete divisor*.

(2) Multiply the complete divisor 28 by 8, the second figure of the root, and subtract the result from the dividend (*c*). The remainder is 24, to which annex the next period making 2,467, as shown at (*e*), which call the *new dividend.*

Add the second figure of the root, or 8, to the divisor 28, and annex a cipher, thus obtaining 360. Dividing 2,467 by 360, we find 6 to be the next figure of the root. Adding this last figure of the root, or 6, to 360, we get 366, which, multiplied by this last figure of the root, or 6, gives 2,196, which we write under 2,467 and subtract.

(3) Annexing the next period 84 to the remainder 271 gives 27,184 as the next new dividend. Now, adding the third figure of the root to 366, and annexing a cipher, as before, we have 3,720. Dividing 27,184 by 3,720, the result is 7, which write as the next figure of the root. Adding the fourth figure of the root, or 7, to 3,720, we get 3,727, which, multiplied by 7 of the root, gives 26,089, which write under 27,184 and subtract, obtaining 1,095 as a remainder.

(4) Annexing the next period, or 40, to the remainder 1,095 gives 109,540 as the next new dividend. Adding the last figure of the root to 3,727, and annexing a cipher, as before, the result is 37,340. Dividing 109,540 by 37,340, the result is 2, which write as the next figure of the root. Adding the fifth figure of the root, or 2, to 37,340, we obtain 37,342, which, multiplied by 2 of the root, gives 74,684, which write under 109,540 and subtract, obtaining 34,856 as a remainder.

(5) Annexing the next period, or 10, to the remainder

34,856 gives 3,485,610 as the next new dividend. Now, adding the last figure of the root to 37,342, and annexing a cipher, as before, the result is 373,440. Dividing 3,485,610 by 373,440 gives 9 as a result, which write as the next, or last figure, of the root. Adding the last figure of the root, or 9, to 373,440, we get 373,449, which, multiplied by 9 of the root, gives 3,361,041, which write under 3,485,610 and subtract. Since there is a remainder, we know the given power is not a perfect square, so we place + after the root.

In this problem there are *six* periods—four in the whole number and two in the decimal—hence, there will be *six* figures in the root (since we obtain one figure of the root for each period), four figures constituting the whole number and two figures the decimal of the root. Hence,

$$\sqrt{3,486,784.4010} = 1,867.29+.$$

$$\begin{array}{lrlr}
(b)\ (a) & 3 & & \sqrt{9'00'00'99.40'09'00} = 3000.016+ \text{ Ans.} \\
 & \underline{3} & (b) & \underline{9\qquad\qquad\qquad} \\
(d) & 60 & (c) & 0000994009 \\
 & \underline{0} & & \underline{600001} \\
 & 600 & & 39400800 \\
 & \underline{0} & & \underline{36000156} \\
 & 6000 & & 3400644 \\
 & \underline{0} & & \\
 & 60000 & & \\
 & \underline{0} & & \\
 & 600000 & & \\
 & \underline{1} & & \\
 & 600001 & & \\
 & \underline{1} & & \\
 & 6000020 & & \\
 & \underline{6} & & \\
 & 6000026 & &
\end{array}$$

Explanation.—Beginning at the decimal point, we point off the whole number into periods of *two* figures each, proceeding from *right* to *left*; also, point off the decimals into periods of *two* figures each, proceeding from *left* to *right*. The largest number whose square is contained in the first period, 9, is 3; hence, 3 is the first figure of the root. Place 3 at the left, as shown at (*a*), and multiply it by the first figure in the root, or 3. The result is 9. Write 9 under the first period, 9, as at (*b*), subtract, and there is no remainder. Bring down the next period, which is 00, as shown at (*c*). Add the root already found to the 3 at (*a*), obtaining 6, and annex a cipher to this 6, thus making it 60, which is the *trial divisor*, as shown at (*d*). Divide the dividend (*c*) by the trial divisor and obtain 0 as the next figure in the root. Write 0 in the root as shown, and also add it to the trial divisor 60, and annex a cipher, thereby making the next trial divisor 600. Bring down the next period, 00, annex it to the dividend already obtained and divide it by the trial divisor. 600 is contained in 0000, 0 times, so we place another cipher in the root. Write 0 in the root, as shown, and also add it to the trial divisor 600, and annex a cipher, thereby making the next trial divisor 6,000. Bring down the next period, 99. The trial divisor 6,000 is contained in 000099, 0 times, so we place 0 as the next figure in the root, as shown, and also add it to the trial divisor 6,000, and annex a cipher, thereby making the next trial divisor 60,000. Bring down the next period, 40, and annex it to the dividend already obtained to form the new dividend, 00009940, and divide it by the trial divisor 60,000. 60,000 is contained in 00009940, 0 times, so we place another cipher in the root, as shown, and also add it to the trial divisor 60,000, and annex one cipher, thereby making the next trial divisor 600,000. Bring down the next period, 09, and annex it to the dividend already obtained to form the new dividend, 0000994009, and divide it by the trial divisor 600,000. 600,000 is contained in 0000994009 once, so we place 1 as the next figure in the root, and also add it to the trial divisor 600,000, thereby making the complete divisor 600,001. Multiply the complete divisor 600,001 by 1, the sixth figure in the root, and subtract the result obtained from the dividend. The remainder is 394,008, to which we annex the next period, 00, to form the next new dividend, or 39,400,800. Add the sixth figure of the root, or 1, to the divisor 600,001, and annex a cipher, thus obtaining 6,000,020 as the next trial divisor. Dividing 39,400,800 by 6,000,020, we find 6 to be the next figure of the root. Adding this last figure, 6, to the trial divisor, we obtain 6,000,026 for our next complete divisor, which, multiplied by the last figure of the root, or 6, gives 36,000,156, which write under 39,400,800 and subtract. Since there is a remainder, it is clearly evident that the given power is not a perfect square, so we place + after the root.

In this problem, there are *seven* periods—four in the whole number and three in the decimal; hence, there will be *seven* figures in the root, *four* figures constituting the whole number and three figures the decimal of the root. Hence, $\sqrt{9{,}000{,}099.4009} = 3{,}000.016+$. Ans.

(*c*)

$$\begin{array}{r} 3\\ 3\\ \hline 60\\ 5\\ \hline 65 \end{array} \qquad \begin{array}{r} \sqrt{.00'12'25} = .035.\ \text{Ans.}\\ 00\phantom{'12'25 = .035.\ \text{Ans.}}\\ \hline 12\phantom{'25 = .035.\ \text{Ans.}}\\ 9\phantom{'25 = .035.\ \text{Ans.}}\\ \hline 325\phantom{= .035.\ \text{Ans.}}\\ 325\phantom{= .035.\ \text{Ans.}}\\ \hline \end{array}$$

Pointing of periods, we find that the first period is composed of ciphers; hence, the first figure of the root will be a cipher. No further explanation is necessary, since this problem is solved in a manner exactly similar to the problem given in Art. **255.** Since there are *three* decimal periods in the power, there will be three decimal figures in the root.

(*d*)

$$\begin{array}{r} 1\\ 1\\ \hline 20\\ 0\\ \hline 200\\ 3\\ \hline 203\\ 3\\ \hline 2060\\ 9\\ \hline 2069 \end{array} \qquad \begin{array}{l} \sqrt{1'07'95.21} = 103.9\quad \text{Ans.}\\ 1\\ \hline 0795\\ 609\\ \hline 18621\\ 18621\\ \hline \end{array}$$

(*e*)

$$\begin{array}{r} 2\\ \underline{2}\\ 40\\ \underline{7}\\ 47\\ \underline{7}\\ 5400\\ \underline{2}\\ 5402 \end{array} \qquad \begin{array}{r} \sqrt{7'30'08.05} = 270.2+ \text{ Ans.}\\ \underline{4}\quad\quad\quad\quad\quad\quad\quad\\ 330\quad\quad\quad\quad\quad\quad\\ \underline{329}\quad\quad\quad\quad\quad\quad\\ 10805\quad\quad\quad\quad\\ \underline{10804}\quad\quad\quad\quad\\ 1\quad\quad\quad\quad \end{array}$$

(*f*) $\sqrt{9} = 3$. Ans.

(*g*)

$$\begin{array}{r} 9\\ \underline{9}\\ 180\\ \underline{4}\\ 184\\ \underline{4}\\ 1880\\ \underline{8}\\ 1888 \end{array} \qquad \begin{array}{r} \sqrt{.90'00'00} = .948+ \text{ Ans.}\\ \underline{81}\quad\quad\quad\quad\quad\quad\quad\\ 900\quad\quad\quad\quad\quad\quad\\ \underline{736}\quad\quad\quad\quad\quad\quad\\ 16400\quad\quad\quad\quad\quad\\ \underline{15104}\quad\quad\quad\quad\quad\\ 1296\quad\quad\quad\quad\quad \end{array}$$

In Art. **258,** we see that every period of a decimal must consist of two figures when extracting the square root. We must, therefore, annex one cipher to .9 to form the first period.

(118) (*a*)

$$\begin{array}{r} (1)\\ 6\\ \underline{6}\\ 12\\ \underline{6}\\ 180\\ \underline{8}\\ 188\\ \underline{8}\\ 196\\ \underline{8}\\ 2040\\ \underline{9}\\ 2049 \end{array} \qquad \begin{array}{r} (2)\\ 36\\ \underline{72}\\ 10800\\ \underline{1504}\\ 12304\\ \underline{1568}\\ 1387200\\ \underline{18441}\\ 1405641 \end{array} \qquad \begin{array}{r} \sqrt[3]{.327'680'000} = .689+ \text{ Ans.}\\ \underline{216}\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\\ 111680\quad\quad\quad\quad\quad\quad\quad\quad\\ \underline{98432}\quad\quad\quad\quad\quad\quad\quad\quad\\ 13248000\quad\quad\quad\quad\quad\quad\\ \underline{12650769}\quad\quad\quad\quad\quad\quad\\ 597231\quad\quad\quad\quad\quad\quad \end{array}$$

EXPLANATION.—(1) When extracting the *cube* root, we divide the power into periods of three figures each. Always begin at the decimal point and proceed to the *left* in pointing off the whole number, and to the *right* in pointing off the decimal. In this power, $\sqrt[3]{327.68}$, a cipher must be annexed to 68 to complete the decimal period. Cipher periods may now be annexed until the root has as many figures as desired.

(2) We find by trial that the largest number whose cube is contained in the first period 327 is 6. Write 6 as the first figure of the root, also at the extreme left at the head of column (1). Multiply the 6 in column (1) by the first figure of the root 6, and write the product 36 at the head of column (2). Multiply the number in column (2) by the first figure of the root 6, and write the product 216 under the figures in the first period. Subtract and bring down the next period 680, annexing it to the remainder 111, thereby obtaining 111,680 for a new dividend. Add the first figure of the root 6 to the number in column (1), obtaining 12, which we call the *first correction;* multiply the first correction 12 by the first figure of the root and we obtain 72 as the product, which, added to 36 of column (2), gives 108. Add the first figure of the root to the first correction, and we obtain 18 as the *second correction.* To this annex *one* cipher, and annexing two ciphers to 108, we have 10,800 for the trial divisor. Dividing the dividend by the trial divisor, we see that it is contained about 8 times, so we write 8 as the second figure of the root. Adding the second figure of the root to 180, we obtain 188. This, multiplied by the second figure of the root 8, equals 1,504, which, added to the trial divisor 10,800, forms the *complete divisor*, 12,304. Multiplying the complete divisor 12,304 by 8, the second figure of the root, gives 98,432. Write 98,432 under the dividend 111,680; subtract, and there is a remainder of 13,248. To this remainder annex the next period, 000, thereby obtaining 13,248,000 for the next new dividend.

(3) Adding the second figure of the root 8 to the number in column (1), 188, we have 196 for the *first new correction.* This, multiplied by the second figure of the root 8, gives 1,568. Adding this product to the last complete divisor gives 13,872. Adding the second figure of the root, 8, to the first new correction, 196, we obtain 204 for the *second new correction.* Annexing one cipher to 204 and two ciphers to 13,872, we obtain 1,387,200 for the new trial divisor. Dividing the dividend by the trial divisor 1,387,200, we see that it is contained about 9 times. Write 9 as the third figure of the root. Add the third figure of the root, 9, to the last number in column (1), 2,040, thereby obtaining 2,049. This, multiplied by 9, the third figure of the root, equals 18,441, which, added to the trial divisor 1,387,200, forms the complete divisor 1,405,641. Multiplying the complete divisor by the third figure of the root 9, and subtracting, we have a remainder of 597,231. The answer, then, is .689 +. There are as many decimal places in the root as there are decimal periods in the power, or three decimal places, in this case.

(*b*)

(1)	(2)
4	16
4	32
8	4800
4	244
120	5044
2	
122	

$\sqrt[3]{74'088} = 42$ Ans.

64

10088

10088

(*c*)

$$\begin{array}{r}
(1)\\
4\\
4\\ \hline
8\\
4\\ \hline
120\\
5\\ \hline
125\\
5\\ \hline
130\\
5\\ \hline
1350\\
2\\ \hline
1352\\
2\\ \hline
1354\\
2\\ \hline
13560\\
1\\ \hline
13561\\
1\\ \hline
13562\\
1\\ \hline
135630\\
1\\ \hline
135631\\
1\\ \hline
135632\\
1\\ \hline
1356330\\
5\\ \hline
1356335
\end{array}
\qquad
\begin{array}{r}
(2)\\
16\\
32\\ \hline
4800\\
625\\ \hline
5425\\
650\\ \hline
607500\\
2704\\ \hline
610204\\
2708\\ \hline
61291200\\
13561\\ \hline
61304761\\
13562\\ \hline
6131832300\\
135631\\ \hline
6131967931\\
135632\\ \hline
613210356300\\
6781675\\ \hline
613217137975
\end{array}$$

45.2115+ Ans.

$$\begin{array}{r}
\sqrt[3]{92'416.000'000'000'000} =\\
64\\ \hline
28416\\
27125\\ \hline
1291000\\
1220408\\ \hline
70592000\\
61304761\\ \hline
9287239000\\
6131967931\\ \hline
3155271069000\\
3066085689875\\ \hline
89185379125
\end{array}$$

Since *four cipher periods* were annexed, we must point off *four* decimal figures in the root.

(*d*)

$\sqrt[3]{.373'248} = .72$ Ans.

343

30248

30248

(1)

7

7

14

7

210

2

212

(2)

49

98

14700

424

15124

(*e*)

$\sqrt[3]{1'758.416'743} = 12.07$ Ans.

1

758

728

30416743

30416743

(1)

1

1

2

1

30

2

32

2

34

2

3600

7

3607

(2)

1

2

300

64

364

68

4320000

25249

4345249

(*f*)

$\sqrt[3]{1'191'016} = 106$ Ans.

1

191016

191016

(1)

1

1

2

1

300

6

306

(2)

1

2

30000

1836

31836

(*g*) In Arts. **279** and **280**, we find that when the given number is in the form of a fraction, and it is required to find some root of it, the simplest method is to reduce the fraction to its equivalent decimal and then extract the required root of the decimal.

Thus, $\sqrt[3]{\frac{4}{32}} = ?$ $\frac{4}{32} = .125$, since

```
32 ) 4.000 ( .125
     32
     --
      80
      64
      ---
      160
      160
      ---
```

```
(1)     (2)     ∛.125 = .5.
 5      25        125
                  ---
```

Hence, $\sqrt[3]{\frac{4}{32}} = .5$. Ans.

(*h*) (See Art. **279.**)

$\sqrt[3]{\frac{27}{256}} = ?$ $\frac{27}{256} = .10546875$, since

```
256 ) 27.00000000 ( .10546875
      256
      ---
       1400
       1280
       ----
        1200
        1024
        ----
         1760
         1536
         ----
          2240
          2048
          ----
           1920
           1792
           ----
            1280
            1280
            ----
```

(1)
4
4
8
4
120
7
127
7
134
7
1410
2
1412

(2)
16
32
4800
889
5689
938
662700
2824
665524

$\sqrt[3]{.105'468'750} = .472+$ Ans.
64
41468
39823
1645750
1331048
314702

(119)

(1)
1
1
2
1
30
2
32
2
34
2
360
5
365
5
370
5
3750
9
3759
9
3768
9
37770
9
37779

(2)
1
2
300
64
364
68
43200
1825
45025
1850
4687500
33831
4721331
33912
475524300
340011
475864311

$\sqrt[3]{2.000'000'000'000} = 1.2599+$
Ans.
1
1000
728
272000
225125
46875000
42491979
4383021000
4282771599
100249401

In this problem it was necessary to annex *four cipher periods*, in order to carry the decimal of the root to the required number of decimal places, or four.

(120)

(1)	(2)	$\sqrt[3]{3.000'000'000} = 1.442+$ Ans.
1	1	1
1	2	2000
2	300	1744
1	136	256000
30	436	241984
4	152	14016000
34	58800	12458888
4	1696	1557112
38	60496	
4	1712	
420	6220800	
4	8644	
424	6229444	
4		
428		
4		
4320		
2		
4322		

In this problem it was necessary to annex *three* cipher periods, in order to carry the decimal of the root to the required number of decimal places, or three.

(121)

(*a*)		(*b*)	
1	$\sqrt{1'23.21} = 11.1$ Ans.	1	$\sqrt{1'14.92'10} = 10.72+$ Ans.
1	1	1	1
20	23	200	1492
1	21	7	1449
21	221	207	4310
1	221	7	4284
220		2140	26
1		2	
221		2142	

(*c*)

7	$\sqrt{50'26'81} = 709$ Ans.
7	49
140	12681
0	12681
1400	
9	
1409	

(*d*)

2	$\sqrt{.00'04'12'09} = .0203$ Ans.
2	00
400	04
3	4
403	1209
	1209

(122) (*a*)

$\sqrt[3]{.006'500'000} = .186+$ Ans.

(1)	(2)	
1	1	1
1	2	5500
2	300	4832
1	304	668000
30	604	602856
8	368	65144
38	97200	
8	3276	
46	100476	
8		
540		
6		
546		

(*b*)

$\sqrt[3]{.021'000} = .27+$ Ans.

(1)	(2)	
2	4	8
2	8	13000
4	1200	11683
2	469	1317
60	1669	
7		
67		

(*c*) $\sqrt[3]{8'036'054'027} = 2003$ Ans

(1)
2
2
4
2
6000
3
6003

(2)
4
8
12000000
18009
12018009

8
036054027
36054027

(*d*) $\sqrt[3]{.000'004'096} = .016$ Ans.

(1)
1
1
2
1
30
6
36

(2)
1
2
300
216
516

000
004
1
3096
3096

(*e*) $\sqrt[3]{17.000'000} = 2.57+$ Ans.

(1)
2
2
4
2
60
5
65
5
70
5
750
7
757

(2)
4
8
1200
325
1525
350
187500
5299
192799

8
9000
7625
1375000
1349593
25407

(123) (*a*) $\sqrt{\frac{1,225}{5,476}} = \frac{\sqrt{1,225}}{\sqrt{5,476}}$.

$$\begin{array}{r} 3\\ 3\\ \hline 60\\ 5\\ \hline 65 \end{array} \qquad \begin{array}{r} \sqrt{12'25} = 35\\ 9\\ \hline 325\\ 325\\ \hline \end{array}$$

$$\begin{array}{r} 7\\ 7\\ \hline 140\\ 4\\ \hline 144 \end{array} \qquad \begin{array}{r} \sqrt{54'76} = 74\\ 49\\ \hline 576\\ 576\\ \hline \end{array}$$

Hence, $\sqrt{\frac{1,225}{5,476}} = \frac{35}{74}$. Ans.

(*b*)

$$\begin{array}{r} 5\\ 5\\ \hline 100\\ 8\\ \hline 108 \end{array} \qquad \begin{array}{r} \sqrt{.33'64} = .58\\ 25 \quad \text{Ans.}\\ \hline 864\\ 864\\ \hline \end{array}$$

(*c*)

$$\begin{array}{r} 3\\ 3\\ \hline 60\\ 1\\ \hline 61\\ 1\\ \hline 620\\ 6\\ \hline 626\\ 6\\ \hline 6320\\ 2\\ \hline 6322\\ 2\\ \hline 63240\\ 2\\ \hline 63242 \end{array} \qquad \begin{array}{r} \sqrt{.10'00'00'00'00} = .31622+\\ 9 \quad \text{Ans.}\\ \hline 100\\ 61\\ \hline 3900\\ 3756\\ \hline 14400\\ 12644\\ \hline 175600\\ 126484\\ \hline 49116 \end{array}$$

(*c*) $\sqrt[3]{8'036'054'027} = 2003$ Ans

(1)
2
2
4
2
6000
3
6003

(2)
4
8
12000000
18009
12018009

8
036054027
36054027

(*d*) $\sqrt[3]{.000'004'096} = .016$ Ans.

(1)
1
1
2
1
30
6
36

(2)
1
2
300
216
516

000
004
1
3096
3096

(*e*) $\sqrt[3]{17.000'000} = 2.57+$ Ans.

(1)
2
2
4
2
60
5
65
5
70
5
750
7
757

(2)
4
8
1200
325
1525
350
187500
5299
192799

8
9000
7625
1375000
1349593
25407

(123) (*a*) $\sqrt{\frac{1,225}{5,476}} = \frac{\sqrt{1,225}}{\sqrt{5,476}}$.

3	$\sqrt{1\,2'2\,5} = 35$
3	9
60	325
5	325
65	

7	$\sqrt{5\,4'7\,6} = 74$
7	49
140	576
4	576
144	

Hence, $\sqrt{\frac{1,225}{5,476}} = \frac{35}{74}$. Ans.

(*b*)

5	$\sqrt{.3\,3'6\,4} = .58$
5	25 Ans.
100	864
8	864
108	

(*c*)

3	$\sqrt{.1\,0'0\,0'0\,0'0\,0'0\,0} = .31622+$
3	9 Ans.
60	100
1	61
61	3900
1	3756
620	14400
6	12644
626	175600
6	126484
6320	49116
2	
6322	
2	
63240	
2	
63242	

(*c*) $\frac{2}{10}=\frac{x}{100}$ is equivalent to $2 : 10 :: x = 100$.

$$\begin{aligned} 10\,x &= 2 \times 100 \\ 10\,x &= 200 \\ x &= \frac{200}{10} = 20. \quad \text{Ans.} \end{aligned}$$

(*d*) $\frac{15}{45}=\frac{60}{x}$ is equivalent to $15 : 45 :: 60 : x$

$$\begin{aligned} 15\,x &= 45 \times 60 \\ 15\,x &= 2{,}700 \\ x &= \frac{2{,}700}{15} = 180. \quad \text{Ans.} \end{aligned}$$

(*e*) $\frac{10}{150}=\frac{x}{600}$ is equivalent to $10 : 150 :: x : 600$.

$$\begin{aligned} 150\,x &= 10 \times 600 \\ 150\,x &= 6{,}000 \\ x &= \frac{6{,}000}{150} = 40. \quad \text{Ans.} \end{aligned}$$

(127) $x : 5 :: 27 : 12.5$

$$\begin{array}{r} 5 \\ \hline 12.5\,)\,135.0\,(\,10\frac{4}{5}.\ \text{Ans.} \\ 125 \\ \hline \frac{100}{125}=\frac{4}{5}. \end{array}$$

Hence, $x = 10\frac{4}{5}$. Ans.

(128) $45 : 60 :: x : 24$

$$\begin{aligned} 60\,x &= 45 \times 24 \\ 60\,x &= 1{,}080 \\ x &= \frac{1{,}080}{60} = 18. \quad \text{Ans.} \end{aligned}$$

(129) $x : 35 :: 4 : 7$

$$\begin{aligned} 7\,x &= 35 \times 4 \\ 7\,x &= 140 \\ x &= \frac{140}{7} = 20. \quad \text{Ans.} \end{aligned}$$

(130) $9 : x :: 6 : 24$

$$\begin{aligned} 6\,x &= 9 \times 24 \\ 6\,x &= 216 \\ x &= \frac{216}{6} = 36. \quad \text{Ans.} \end{aligned}$$

(131) $\sqrt[3]{1,000} : \sqrt[3]{1,331} :: 27 : x$ $\qquad \sqrt[3]{1,000} = 10$

$10 : 11 :: 27 : x$ $\qquad \sqrt[3]{1,331} = 11$

$10x = 297$

$x = \frac{297}{10} = 29.7.$ Ans.

$$\begin{array}{lll}
(1) & (2) & \\
1 & 1 & 1'331(11 \\
\underline{1} & \underline{2} & \underline{1} \\
2 & 300 & 331 \\
\underline{1} & \underline{31} & \underline{331} \\
30 & 331 & \\
\underline{1} & & \\
31 & &
\end{array}$$

(132) $64 : 81 :: 21^2 : x^2$. Extracting the square root of each term of any proportion does not change its value, so we find that $\sqrt{64} : \sqrt{81} :: \sqrt{21^2} : \sqrt{x^2}$ is the same as

$$8 : 9 :: 21 : x$$
$$8x = 189$$
$$x = 23.625. \quad \text{Ans.}$$

(133) $7 + 8 : 7 :: 30 : x$ is equivalent to $15 : 7 :: 30 : x$.

$$15x = 7 \times 30$$
$$15x = 210$$
$$x = \frac{210}{15} = 14. \quad \text{Ans.}$$

(134) $3\frac{1}{2}$ ft. = 3.5 ft., since $\frac{1}{2} = .5$.

$6\frac{3}{4}$ ft. = 6.75 ft., since $\frac{3}{4} = .75$.

Consider the question "What do we wish to find?" In this case it is "pounds." We know that a piece of shafting 3.5 ft. long weighs 37.45 lb., and we wish to know the weight of a piece of shafting $6\frac{3}{4}$ ft. long. It is evident that the weight of a piece of shafting 6.75 ft. long bears the same relation to the weight of a piece 3.5 ft. long that 6.75 ft. bears to 3.5 ft. Letting x occupy any place in the proportion, we have the following, the value of x being the same in each. Thus,

(*a*) 3.5 ft. : 6.75 ft. :: 37.45 lb. : x lb.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(*b*) 6.75 ft. : 3.5 ft. :: x lb. : 37.45 lb.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(*c*) x lb. : 37.45 lb. :: 6.75 ft. : 3.5 ft.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(*d*) 37.45 lb. : x lb. :: 3.5 ft. : 6.75 ft.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(135) In this problem we are required to find "degrees." In examples of this kind, it is better to write the statement down as a simple direct proportion, and then make what changes are necessary. Written as a direct proportion, we would have $6 : 12 :: 24 : x$. But, since the heat decreases *inversely*, the proportion is an inverse one. Hence, inverting one of the couplets, we have $12 : 6 :: 24 : x$. The example also states that the effect of the heat upon the thermometer decreases inversely as the *square of the distance* from the burning body. Consequently, the two distances, 6 ft. and 12 ft., in the above proportion must be squared, and we have $12^2 : 6^2 :: 24 : x$. $12^2 = 144$ and $6^2 = 36$. Therefore, $144 : 36 :: 24 : x$,

$$\text{or } x = \frac{36 \times 24}{144} = 6°. \text{ Ans.}$$

(136) This problem may be solved by very simple reasoning, thus: If sound travels at the rate of 6,160 feet in $5\frac{1}{2}$ seconds, in 1 minute, which is $10\frac{10}{11}$ times $5\frac{1}{2}$ seconds

$\left(60 \div \frac{11}{2} = 60 \times \frac{2}{11} = \frac{120}{11} = 10\frac{10}{11}\right)$, it will travel $10\frac{10}{11} \times$ 6,160 ft., or 67,200 ft. Ans.

$$\begin{array}{r} 6160 \\ 10\frac{10}{11} \\ \hline 61600 \\ 5600 \\ \hline 67200 \text{ ft. Ans.} \end{array} \qquad \frac{10}{11} \times 6{,}160 = \frac{61{,}600}{11} = 5{,}600.$$

Or, if sound travels at the rate of 6,160 feet in $5\frac{1}{2}$ seconds, in 1 second it will travel as many feet as $5\frac{1}{2}$ (5.5) is contained times in 6,160 feet, or 1,120 feet, and in 1 minute, or 60 seconds, it will travel 1,120 × 60, or 67,200 feet.

Or, it may be solved by proportion, as follows: In this problem it states that sound travels at the rate of 6,160 feet in $5\frac{1}{2}$ (5.5) seconds. We wish to know how far it will travel in 1 minute, which is equivalent to 60 seconds. It is evident that the distance it travels in 1 minute, or 60 seconds, bears the same relation to the distance it travels in 5.5 seconds that 1 minute, or 60 seconds, bears to 5.5 seconds. Letting x occupy any place in the proportion, we have the following, the value of x being the same in each. Thus,

(*a*) 6,160 feet : x feet :: 5.5 seconds : 60 seconds,

$$\text{or } x = \frac{6{,}160 \times 60}{5.5} = \frac{369{,}600}{5.5} \qquad 5.5)\,369600.0\,(67200 \text{ ft. Ans.}$$

$$\begin{array}{r} 330 \\ \hline 396 \\ 385 \\ \hline 110 \\ 110 \\ \hline \end{array}$$

(*b*) x feet : 6,160 feet :: 60 seconds : 5.5 seconds,

$$\text{or } x = \frac{6{,}160 \times 60}{5.5} = 67{,}200 \text{ feet. Ans.}$$

(*c*) 60 seconds : 5.5 seconds :: x feet : 6,160 feet,

$$\text{or } x = \frac{6,160 \times 60}{5.5} = 67,200 \text{ feet. Ans.}$$

(*d*) 5.5 seconds : 60 seconds :: 6,160 feet : x feet,

$$\text{or } x = \frac{6,160 \times 60}{5.5} = 67,200 \text{ feet. Ans.}$$

(137) We will first reduce 8 hr. 40 min. to minutes. 8 hr. + 40 min. = (8 × 60 min.) + 40 min. = 520 min. In this problem we are required to find "time." We know that a railway train runs 444 miles in 520 minutes, and we want to know how long it will take it to run 1,060 miles at the same rate of speed. It is evident that the time it requires to run 1,060 miles bears the same relation to the time it takes to run 444 miles that 1,060 miles bears to 444 miles. Letting x occupy any place in the proportion, we have the following, the value of x being the same in each. Thus,

(*a*) 1,060 miles : 444 miles :: x min. : 520 min.,

$$\text{or } x = \frac{1,060 \times 520}{444} = \frac{551,200}{444} =$$

```
  1060        444)551200.00(1241.44+ min.
   520            444
 ------           ----
 21200            1072
5300               888
 ------            ----
551200             1840
                   1776
                   ----
                    640
                    444
                   ----
                    1960
                    1776
                    ----
                     1840
                     1776
                     ----
                       64
```

Reducing 1,241.44 min. to hours by dividing by 60, we have

$$\begin{array}{l} 60)1241.44(20 \text{ hr. } 41.44 \text{ min. Ans.} \\ \quad\;\; \underline{120} \\ \qquad\; 41 \end{array}$$

(*b*) 444 miles : 1,060 miles :: 520 min. : x min.

$$x = \frac{1,060 \times 520}{444} = 1,241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min. Ans.}$$

(*c*) x min. : 520 min. :: 1,060 miles : 444 miles.

$$x = \frac{1,060 \times 520}{444} = 1,241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min. Ans.}$$

(*d*) 520 min. : x min. :: 444 miles : 1,060 miles.

$$x = \frac{1,060 \times 520}{444} = 1,241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min. Ans.}$$

(138) This is an *inverse* ratio since it will take a pump discharging 85 gal. per minute a *longer* time to fill the tank than one discharging 135 gal. per minute. The direct proportion would be

135 gal. : 85 gal. :: 38 min. : x min.

In order to make the proportion inverse, we must invert one of the couplets, and we have

135 gal. : 85 gal. : x min. : 38 min.

$$x = \frac{135 \times 38}{85} = 60\frac{6}{17} \text{ min. Ans.}$$

$$\begin{array}{r} 135 \\ \times\; 38 \\ \hline 1080 \\ 405 \\ \hline 85)5130(60\frac{6}{17} \\ 510 \\ \hline \frac{30}{85} = \frac{6}{17} \end{array}$$

(139) 8 lb. + 8 lb. + 80 lb. = 96 lb., or the number of pounds in the mixture. The relation between the amount of copper which 36 lb. of this mixture will contain, and the amount which 96 lb. contain is the same as 32 : 96; whence, the proportion,

$$32 \text{ lb.} : 96 \text{ lb.} :: x : 8 \text{ lb.},$$

$$\begin{array}{l} 96)256(2 \\ \quad\underline{192} \\ \quad\ \ \frac{64}{96}=\frac{2}{3} \end{array} \qquad \text{or } x = \frac{32 \times 8}{96} = \frac{256}{96} = 2\frac{2}{3} \text{ lb. Ans.}$$

(140) This will be an *inverse* ratio, since the *larger* wheel will turn a less number of times than the smaller wheel in going a certain distance. The direct proportion is

$$12.56 \text{ ft.} : 15.7 \text{ ft.} :: 520 : x.$$

Inverting one of the couplets,

$$15.7 \text{ ft.} : 12.56 \text{ ft.} :: 520 : x,$$

$$\text{or } x = \frac{12.56 \times 520}{15.7} = 416 \text{ times. Ans.}$$

$$\begin{array}{r} 520 \\ \underline{12.56} \\ 3120 \\ 2600 \\ 1040 \\ \underline{520} \end{array}$$

$$\begin{array}{l} 15.7\,)\,6531.20\,(\,416 \text{ times} \\ \qquad\ \underline{628} \\ \qquad\quad \ 251 \\ \qquad\quad \underline{\ 157} \\ \qquad\qquad\ 942 \\ \qquad\qquad\ \underline{942} \end{array}$$

(141) Before forming the proportion, we will combine the three simple ratios into one by reducing them all to the same denomination. A cistern 28 feet long, 12 feet wide, and 10 feet deep contains 28 × 12 × 10 = 3,360 cubic feet.

Again, a cistern 20 ft. long, 17 ft. wide, and 6 feet deep contains $20 \times 17 \times 6 = 2,040$ cubic feet. What do we wish to find? In this case it is "barrels." We know that a cistern containing 3,360 cubic feet holds 510 barrels of water, and we want to know how many barrels of water a cistern containing 2,040 cubic feet will hold. The number of barrels that a cistern containing 2,040 cubic feet will hold bears the same relation to the number of barrels that a cistern containing 3,360 cubic feet holds as 2,040 cubic feet bears to 3,360 cubic feet. Hence,

$$2,040 \text{ cu. ft.} : 3,360 \text{ cu. ft.} :: x \text{ bbl.} : 510 \text{ bbl.},$$

$$\text{or} \quad = \frac{\overset{85}{\cancel{2},\cancel{0}\cancel{4}\cancel{0}} \times 51\cancel{0}}{\underset{14}{\cancel{3},\cancel{3}\cancel{6}\cancel{0}}} = \frac{4,335}{14} = 309\frac{9}{14} \text{ bbl.} \quad \text{Ans.}$$

MENSURATION AND USE OF LETTERS IN ALGEBRAIC FORMULAS.

(QUESTIONS 142–213.)

(142) Substituting for D, x, B, and i their values,

$$C = \frac{D - x}{B + i} = \frac{120 - 12}{10 + 3.5} = \frac{108}{13.5} = 8. \quad \text{Ans.}$$

A line between two numbers signifies that the one above the line, or numerator, is to be divided by the one below the line, or denominator.

(143) Substituting for A, h, D, and x their values,

$$\frac{A\,h + D}{2x + 6} = \frac{(5 \times 200) + 120}{(2 \times 12) + 6} = \frac{1{,}000 + 120}{24 + 6} = \frac{1{,}120}{30} = 37\tfrac{1}{3}.$$

$$37\tfrac{1}{3} + D = 37\tfrac{1}{3} + 120 = 157\tfrac{1}{3}. \quad \text{Ans.}$$

When there is no sign between the letters, multiplication is understood.

(144) Substituting for B, h, A, x, and i their values,

$$r = \frac{3.246 \times B \times h}{\dfrac{A\,x + h}{A\,i - B}} = \frac{3.246 \times 10 \times 200}{\dfrac{(5 \times 12) + 200}{(5 \times 3.5) - 10}} = \frac{6{,}492}{\dfrac{260}{7.5}} =$$

$$6{,}492 \div \frac{260}{7.5} = 6{,}492 \times \frac{7.5}{260} = 187.269 +. \quad \text{Ans.}$$

(145) Substituting for A, D, i, and B their values,

$$v = \sqrt{\frac{A\,D}{i\,B + 1.5}} = \sqrt{\frac{5 \times 120}{(3.5 \times 10) + 1.5}} = \sqrt{\frac{600}{36.5}} =$$

$$\sqrt{16.4383} = 4.05 +. \quad \text{Ans.}$$

The square root sign extends over both numerator and denominator, thus indicating that the square root of the entire fraction is to be extracted.

(146) Substituting for B, x, h, and A their values,

$$u = \sqrt[3]{\frac{Bx}{.00018\,h\,(A^2 - x)}} = \sqrt[3]{\frac{10 \times 12}{.00018 \times 200 \times (5^2 - 12)}} =$$

$$\sqrt[3]{\frac{120}{.036 \times (25 - 12)}} = \sqrt[3]{\frac{120}{.036 \times 13}} = \sqrt[3]{\frac{120}{.468}} =$$

$$\sqrt[3]{256.41} + = 6.35 +. \quad \text{Ans.}$$

(147) Substituting for h, D, and A their values,

$$f = \frac{10(h - D)^2}{\sqrt[3]{D + A}} = \frac{10\,(200 - 120)^2}{\sqrt[3]{120 + 5}} = \frac{10 \times 80^2}{\sqrt[3]{125}} = \frac{64{,}000}{5} = 12{,}800. \quad \text{Ans.}$$

(148) Substituting for B, A, and D their values,

$$g = \frac{(B - A)^2 - \sqrt[3]{D + A}}{A^2 - (1 + D)} = \frac{(10 - 5)^2 - \sqrt[3]{120 + 5}}{5^2 - (1 + 120)} =$$

$$\frac{5^2 - \sqrt[3]{125}}{125 - 121} = \frac{25 - 5}{4} = \frac{20}{4} = 5. \quad \text{Ans.}$$

(149) Substituting for A, B, and h their values,

$$k = \sqrt{\frac{AB^2}{\sqrt[3]{Ah}}} = \sqrt{\frac{5 \times 10^2}{\sqrt[3]{5 \times 200}}} = \sqrt{\frac{5 \times 100}{\sqrt[3]{1{,}000}}} = \sqrt{\frac{500}{10}} =$$

$$\sqrt{50} = 7.071 +. \quad \text{Ans.}$$

(150) Substituting for A, h, D, x, and B their values,

$$T = \sqrt{\frac{A^2\left[490 + \frac{(hx)^2}{D^2}\right]}{h + \frac{x}{D}(A^2 - B)^2}} = \sqrt{\frac{5^2\left[490 + \frac{(200 \times 12)^2}{120^2}\right]}{200 + \frac{12}{120}(5^2 - 10)^2}} =$$

$$\sqrt{\frac{25\,(490 + 400)}{200 + (\frac{1}{10} \times 225)}} = \sqrt{\frac{25 \times 890}{200 + 22.5}} = \sqrt{\frac{22{,}250}{222.5}} =$$

$$\sqrt{100} = 10. \quad \text{Ans.}$$

(151) When one straight line meets another straight line, two angles are formed which together equal 180°. Hence, if one of the angles = 152° 3′, the other angle = 180° − 152° 3′, or

$$\begin{array}{rr} & 180^\circ = 179^\circ\ 60' \\ \text{subtracting,} & 152^\circ\ \ 3' \\ \hline & 27^\circ\ 57'. \end{array} \quad \text{Ans.}$$

(152) There are 60 seconds in one minute and 60 minutes in one degree; therefore, $140° = 140 \times 60 = 8,400$ minutes; $8,400' + 17' = 8,417'$; $8,417' = 8,417 \times 60 = 505,020$ seconds, and $505,020'' + 10'' = 505,030''$. Ans.

(153) See Arts. **359** and **360.**

(154) (*a*) $240 \div 60 = 4$, the number of degrees. Ans.

(*b*) $240 \times 60 = 14,400$, the number of seconds. Ans.

(155) See Arts. **355-357.**

(156) See Art. **369.** A rectangle with the same area would have the same base and altitude.

(157) No, since the sum of the three shorter sides is not greater than the fourth side.

(158) Since the area is to be found in square inches, the $2\frac{1}{2}$ feet must be reduced to inches. $2\frac{1}{2}$ ft. = 30 in. Area $= 30 \times 11\frac{1}{2} = 345$ sq. in. Ans.

(159) Since there are 144 sq. in. in 1 sq. ft., the area of the zinc in the last example $= \frac{345}{144} = 2.396$ sq. ft. The weight per square foot $= \frac{5.25}{2.396} = 2.19 +$ lb. Ans.

(160) It will take $1\frac{1}{2}$ boards to reach lengthways of the room. Since the room is 15 feet wide, and each board is 5 *inches* wide, it will take $15 \div \frac{5}{12} = 36$ boards, laid side by side, to extend across the width of the room. Hence, number of boards required $= 36 \times 1\frac{1}{2} = 54$. Ans.

(161) By the rule for the area of a trapezoid, area of the land $= \frac{9 + 6}{2} \times 16 = 120$ square rods. Since there are 160 square rods in one acre, 120 square rods $= \frac{120}{160} = \frac{3}{4}$ of an acre. Ans.

(162) The total area of the floor of the station $= 55 \times 58$ ft. $= 3,190$ sq. ft. $- 25 \times 26$ ft. $= 650$ sq. ft., the area represented by the lower right-hand corner of the figure. Hence, total area of floor $= 3{,}190 - 650 = 2{,}540$ sq. ft.

From this we have to deduct the following areas:

2 boilers $= 2 \times 8 \times 19 = 304.$ sq. ft.
Feed pump $= 2\frac{1}{2} \times 5 = 12.5$ sq. ft.
2 engines $= 2 \times 4\frac{1}{2} \times 10 = 90$ sq. ft.
2 dynamos $= 2 \times 5\frac{1}{2} \times 6\frac{1}{2} = 71.5$ sq. ft.
Switchboard $= \dfrac{10 \times 3.5}{12} = 2.92$ sq. ft.
480.92 sq. ft.

The unoccupied floor space, therefore, $=$
$2{,}540 - 480.92 = 2{,}059.08$ sq. ft. Ans.

(163) The length of the walk, neglecting the corners, that is, the distance $a\,b + b\,c + c\,d + d\,a$, Fig. 1, $= 2 \times 528 + 2 \times 352 = 1{,}760$ ft. The width $= 10$ ft.; area $= 1{,}760 \times 10 = 17{,}600$ sq. ft. The area of the four corners $= 4 \times 10^2 = 400$ sq. ft. Total area of walk $= 17{,}600 + 400 = 18{,}000$ sq. ft. Hence the area in sq. yds. $= \dfrac{18{,}000}{9} = 2{,}000$. Ans.

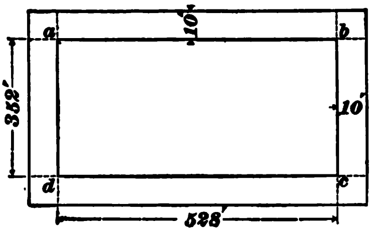

FIG. 1.

(164) The area of the sides of the room $= 2 \times 20 \times 90 = 3{,}600$ sq. ft.; area of ends $= 2 \times 20 \times 50 = 2{,}000$ sq. ft. Total area of walls $= 3{,}600 + 2{,}000 = 5{,}600$ sq. ft.

From this is to be deducted the following areas:

4 doors $= 4 \times 5\frac{1}{2} \times 10 = 220$ sq. ft.
14 windows $= 14 \times 5 \times 11 = 770$ sq. ft.

Length of baseboard, deducting the width of the 4 doors $= 2 \times 90 + 2 \times 50 - 4 \times 5\frac{1}{2} = 180 + 100 - 22 = 258$ feet; width of baseboard $= 9$ inches, or $\frac{3}{4}$ of a foot; area of baseboard $= 258 \times \dfrac{3}{4} = 193.5$ sq. ft. Hence, we have to deduct $220 + 770 + 193.5 = 1{,}183.5$ sq. ft.

Number of sq. ft. of plastering = 5,600 − 1,183.5 = 4,416.5 sq. ft.; number of sq. yd. of plastering = 4,416.5 ÷ 9 = 490.72. Ans.

(165) A triangle with three equal angles has three equal sides, and is, therefore, an equilateral triangle.

(166) A triangle with two equal angles has two equal sides, and is, therefore, an isosceles triangle.

(167) No, since the sum of the two shorter sides is not greater than the third side.

(168) (*a*) Draw a line BD from the vertex perpendicular to the base, Fig. 2. It will divide the base into two equal parts, as shown. In the right-angled triangle ABD, the hypotenuse $AB = 6$, and side $AD = 3$; hence, by rule **50**, Art. **385**, BD, the altitude = $\sqrt{6^2 - 3^2} = \sqrt{27} = 5.196$ ft. Ans.

(*b*) Area $= \frac{6 \times 5.196}{2} = 15.588$ sq. ft. Ans.

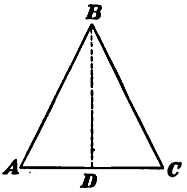

FIG. 2.

(169) The sum of the three angles in any triangle = 2 right angles, or 180°. In the given triangle, the sum of two angles = 23° + 32° 32′ = 55° 32′, and the third angle = 180° − 55° 32′, or

$$\begin{array}{rr} & 180^\circ = 179^\circ\ 60' \\ \text{subtracting,} & 55^\circ\ 32' \\ \hline & 124^\circ\ 28'. \end{array}$$ Ans.

(170) In Fig. 3 we have the proportion $AD : DE :: AB : BC$, in which $AD = 10$ in., $AB = 24$ in., and $BC = 13\frac{1}{2}$ in., to find DE.

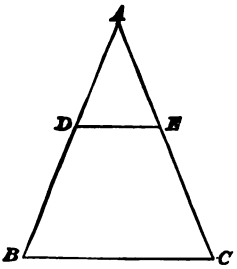

FIG. 3.

Substituting the given values,

$$10 : DE :: 24 : 13\tfrac{1}{2}, \text{ or}$$

$$DE = \frac{10 \times 13.5}{24} = 5.625 \text{ in.} \quad \text{Ans.}$$

(171) A line drawn diagonally from one corner to the opposite one would form the hypotenuse of a right-angled

AB and AC, and the side BC each = 1 inch, and the triangle ABC is equilateral. Draw the line AD perpendicular to the side BC; it will bisect BC. Then, in the right-angled triangle ADB, $AB = 1$, and $BD = \frac{1}{2}''$, to find AD. According to rule **50**, Art. **385**, $AD = \sqrt{1^2 - .5^2} = \sqrt{.75} = .866''$. Hence, the distance between two opposite sides of the hexagon $= AD \times 2 = .866 \times 2 = 1.732''$. Ans.

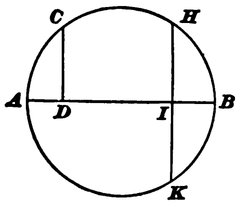

Fig. 7.

(183) In Fig. 7, we have the proportion $BI : HI :: HI : IA$, in which $BI = 6$, and $HI = \frac{1}{2}$ of $HK = \frac{18}{2} = 9$. Substituting, $6 : 9 :: 9 : IA$, or $IA = \frac{81}{6} = 13.5$ in. Hence, the diameter $AB = IA + BI = 13.5 + 6 = 19.5$ in. Ans.

(184) If the diameter $AB = 32\frac{1}{2}$ ft. and $IB = 8$ ft., $AI = 32\frac{1}{2} - 8 = 24\frac{1}{2}$ ft. Then, from the proportion of the last example, $8 : HI :: HI : 24.5$, whence $HI = \sqrt{8 \times 24.5} = \sqrt{196} = 14$ ft., and $HK = 2 \times 14 = 28$ ft. Ans. (See Art. **400.**)

(185) Diameter = circumference ÷ 3.1416. By rule **56,** Art. **403,** diameter of tree $= 7.854 \div 3.1416 = 2\frac{1}{2}$ ft. Ans.

(186) One mile = 5,280 feet. The circumference of the wheel in feet $= \frac{72 \times 3.1416}{12} = 18.8496$. (See rule **55,** Art. **402.**) Number of revolutions in going one mile $= 5{,}280 \div 18.8496 = 280.112$. Ans.

(187) Using rule **58,** Art. **405,** area = diameter squared $\times .7854$. $6.06^2 = 36.7236$; $36.7236 \times .7854 = 28.8427$ sq. in. Ans.

(188) By rule **59,** Art. **406,** inside diameter $= \sqrt{\frac{113.0973}{.7854}} = 12''$. Since the pipe is $\frac{3}{4}''$ thick, the outside diameter must be $\frac{3}{4}'' \times 2 = 1\frac{1}{2}''$ more, or $12 + 1\frac{1}{2} = 13\frac{1}{2}''$. Ans

(189) Since the radius of the circle = 6 in., its diameter = 12 in., and its circumference = 12 × 3.1416 = 37.6992 in. There are 360° in the circumference, and the length of an arc of 12° $= 37.6992 \times \frac{12}{360} = 1.25664$ in. Ans.

(190) The area of a circle 22 inches in diameter $= 22^2 \times .7854 = 380.1336$ sq. in. (See rule **58,** Art. **405.**) Area of a circle 21 inches in diameter $= 21^2 \times .7854 = 346.3614$ sq. in. Hence, the area of a flat ring whose outside diameter = 22 in. and inside diameter = 21 in. is $380.1336 - 346.3614 = 33.7722$ sq. in. Ans.

(191) In the formula of rule **61,** Art. **408,** $\frac{4h^2}{3}\sqrt{\frac{D}{h} - .608}$, h = the height of the segment = 5 in., and D = the diameter of the circle = 56 in. Hence, the area of the segment $= \frac{4 \times 5^2}{3}\sqrt{\frac{56}{5} - .608} = \frac{100}{3}\sqrt{10.592} = 33\frac{1}{3} \times 3.255 =$ 108.5 sq. in. Ans.

(192) In Fig. 8, let $A\,C\,B\,D$ represent a section of the largest square bar that can be planed from the round bar. $A\,B$ and $C\,D$ each = 2 in., and in the right-angled triangle $A\,O\,C$, the sides $A\,O$ and $C\,O$ each = 1 in. The hypotenuse $A\,C = \sqrt{1^2 + 1^2} = \sqrt{2} = 1.4142$ in. Ans. Hence, the largest square bar that can be planed from the round bar is 1.4142″ square.

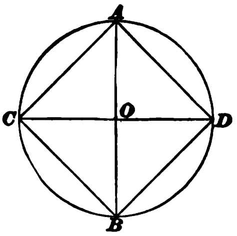

FIG. 8.

(193) The area of a circle 15 in. in diameter $= 15^2 \times .7854 = 176.715$ sq. in. Hence, the area of a sector of this circle, whose angle is $12\frac{1}{2}°$, $= 176.715 \times \frac{12\frac{1}{2}}{360} = \frac{2{,}208.937}{360} =$ 6.1359 sq. in. Ans. (See rule **60,** Art. **407.**)

(194) (*a*) The side of a square whose area = 103.8691 sq. in. $= \sqrt{103.8691} = 10.1916$ in. Ans.

(*b*) By rule **59,** Art. **406,** the diameter of a circle having the same area $= \sqrt{\frac{103.8691}{.7854}} = 11\frac{1}{2}$ in. Ans.

(*c*) Perimeter of the square $= 10.1916 \times 4 = 40.7664$ in.; circumference of the circle $= 11.5 \times 3.1416 = 36.1284$ in.; difference $= 40.7664 - 36.1284 = 4.638$ in. Ans.

(195) With 5 in. allowed for lap, the length of the plate which forms the shell is $46 - 5 = 41$ in. This is the circumference of the shell. By rule **56,** Art. **403,** the diameter corresponding to this circumference is $\frac{41}{3.1416} =$ 13.05 in. Ans.

(196) The convex area is the area of the outside surface, not including the area of the ends. The circumference of the base $= 26 \times 3.1416 = 81.6816$ in. This reduced to feet, since the area is to be in feet, $= 81.6816 \div 12 = 6.8068$. Using rule **62,** Art. **416,** convex area $= 6.8068 \times 10\frac{1}{2} =$ 71.4714 sq. ft. Ans.

(197) The perimeter of the base $= 4 \times 6 = 24$ in. $= 2$ ft. Convex area $= 2 \times 12 = 24$ sq. ft. The area of the bases is found as follows: In Fig. 9, $AB = 4$ in. and $AC = 2$ in.; since this is a regular hexagon, $AO = AB = 4$ in. By rule **50,** Art. **385,** $OC = \sqrt{4^2 - 2^2} = \sqrt{12} = 3.4641$ in.; area of triangle $AOB = \frac{4 \times 3.4641}{2} = 6.9282$ sq. in.; area of base $= 6.9282 \times 6 = 41.5692$, and the area of both bases $= 41.5692 \times 2 = 83.1384$ sq. in. This reduced to square feet $= \frac{83.1384}{144} = .5774$. Hence, the area of the entire surface of the column is $24 + .5774 = 24.5774$ sq. ft. Ans.

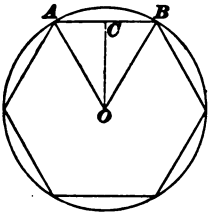

FIG. 9.

(198) The cubical contents in cubic inches = area of base in square inches × altitude in inches. The area of the base in the last example was found to be 41.5692 sq. in.;

altitude $= 12 \times 12 = 144$ in. Hence, the cubical contents $= 41.5692 \times 144 = 5,985.9648$ cu. in. Ans.

(199) This example is solved by combining the rules for the circular ring (see example, Art. **406**) and for the cylinder. To obtain the area of one end of the tube, we have $4^2 \times .7854 = 12.5664 =$ area of a circle 4 inches in diameter; $3.73^2 \times .7854 = 10.9272 =$ area of a circle 3.73 inches in diameter; difference $= 12.5664 - 10.9272 = 1.6392 =$ area of one end of the tube. The cubical contents $= 1.6392 \times 12 = 19.6704$ cu. in.; the weight $= 19.6704 \times .28 = 5.5$, or $5\frac{1}{2}$ lb. Ans.

(200) This example is done exactly like the one in Art. **417**, and the solution is given here without explanation.

(*a*) In the formula of rule **61**, Art. **408**, $\frac{4h^2}{3}\sqrt{\frac{D}{h} - .608}$, h in this case $= 18$, and $D = 60$.

Substituting, area =

$$\frac{4 \times 18^2}{3}\sqrt{\frac{60}{18} - .608} = \frac{4 \times 324}{3}\sqrt{3.333 - .608} = 432 \times \sqrt{2.725} =$$

$432 \times 1.65 = 712.8$ sq. in. This reduced to square feet $= 712.8 \div 144 = 4.95$. Hence, the steam space $= 4.95 \times 16 = 79.2$ cu. ft. Ans.

(*b*) Total area of one end of boiler in square inches $= 60^2 \times .7854 = 2,827.44$. From this is to be subtracted the area of the tube ends and of the segment found above.

Area of ends of tubes $= 3.5^2 \times .7854 \times 64 = 615.75$ sq. in.
Area of segment $=$ 712.8 sq. in.
1,328.55 sq. in.

Area of water space $= 2,827.44 - 1,328.55 = 1,498.89$ sq. in.

Contents of water space $= 1,498.89 \times 16 \times 12 = 287,786.88$ cu. in., and $287,786.88 \div 231 = 1,245.83$, number of gallons, or say 1,246 gal. Ans.

(201) (*a*) Area of piston $= 19^2 \times .7854 = 283.529$ sq. in., or 1.9689 square feet (rule **58**, Art. **405**).

Length of stroke plus the clearance $= 1.14 \times 2$ ft. (24 in. $= 2$ ft.) $= 2.28$ ft.

$1.9689 \times 2.28 = 4.489$ cubic feet, or the volume of steam in the small cylinder (rule **63**, Art. **417**).

(*b*) Area of piston $= 31^2 \times .7854 = 754.7694$ sq. in., or 5.2414 square feet.

Length of stroke plus the clearance $= 1.08 \times 2 = 2.16$ ft.

$5.2414 \times 2.16 = 11.321$ cubic feet, or the volume of steam in the large cylinder. Ans.

(*c*) Ratio $= \dfrac{11.321}{4.489}$ or $2.522 : 1$. Ans.

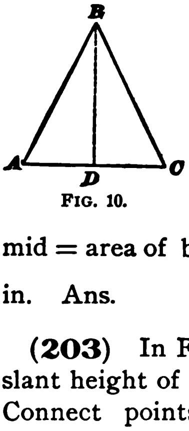

FIG. 10.

(202) Let the equilateral triangle ABC, Fig. 10, represent the base of the pyramid. By rule **50**, Art. **385**, the altitude AD of the triangle $= \sqrt{10^2 - 5^2} = \sqrt{75} = 8.6602$ in., and according to rule **51**, Art. **386**, the area of the triangle $= \dfrac{10 \times 8.6602}{2} = 43.301$ sq. in. Using rule **65**, Art. **423**, volume of pyramid $=$ area of base $\times \dfrac{1}{3}$ altitude $= \dfrac{43.301 \times 10}{3} = 144.336$ cu. in. Ans.

(203) In Fig. 11, let OH be the altitude and OE the slant height of the pyramid. Connect points H and E, forming the right-angled triangle OHE, in which we have to find OH. Since it is a right pyramid, point H will fall at the center of the base $ABCD$, and, hence, the line $HE = \dfrac{1}{2} AB$, or 8 in.; $OE = 25$ in.; by rule **50**, Art. **385**, $OH = \sqrt{25^2 - 8^2} = \sqrt{561} = 23.6854$ in. Ans.

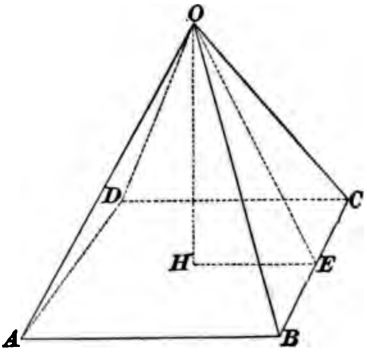

FIG. 11.

(204) The area of the convex surface = circumference of base $\times \frac{1}{2}$ slant height $= 18.8496 \times \frac{10}{2} = 94.248$ sq. in. (See rule **64,** Art. **422.**) The area of the entire surface = 94.248 sq. in. + the area of the base. The diameter of the base $= \frac{18.8496}{3.1416} = 6$ in.; hence, the area of the base $= 6^2 \times .7854 = 28.2744$ (rules **56** and **58,** Arts. **403** and **405**); therefore, the area of the entire surface $= 94.248 + 28.2744 = 122.5224$ sq. in. Ans.

(205) Using rule **65,** Art. **423,** volume = area of base $\times \frac{1}{3}$ altitude $= 28.2744 \times \frac{9}{3} = 84.8232$ cu. in. Ans.

(206) The vat has the form of an inverted frustum of a pyramid. Area of larger base $= 15^2 = 225$ sq. ft.; area of smaller base $= 12^2 = 144$ sq. ft. Hence, by rule **67,** Art. **427,** the contents of the vat in cubic feet $= (225 + 144 + \sqrt{225 \times 144})\frac{11}{3} = (369 + 180) \times \frac{11}{3} = 549 \times \frac{11}{3} =$ 2,013 cu. ft. This should be reduced to cubic inches by multiplying by 1,728, the number of cubic inches in a cubic foot. 2,013 × 1,728 = 3,478,464 cu. in. Since there are 231 cubic inches in a gallon, the number of gallons that the vat will hold $= \frac{3,478,464}{231} = 15,058.29$. Ans.

(207) The pail is in the form of a frustum of a cone. Area of larger base $= 12^2 \times .7854 = 113.0976$ cu. in. Area of smaller base = 63.6174 cu. in. Hence, the contents in cubic inches =

$$(113.0976 + 63.6174 + \sqrt{113.0976 \times 63.6174}) \times \frac{11}{3} =$$

$$(176.715 + \sqrt{7,194.9753})\frac{11}{3} = (176.715 + 84.8232) \times \frac{11}{3} =$$

$$261.5382 \times \frac{11}{3} = 958.9734.$$

The contents of the vat in cubic inches were found in the last example to be 3,478,464. Hence, the number of pails of water required to fill the vat $= 3{,}478{,}464 \div 958.9734 = 3{,}627.28$. Ans.

(208) By rule **66**, Art. **426**, the area of the convex surface = half the sum of the perimeters of the upper and lower bases × the slant height, or $\frac{48 + 36}{2} \times 32 = 42 \times 32 = 1{,}344$ sq. in.

(209) See note following the question.

Outside diameter of upper base $= \frac{170.5}{3.1416} =$ about 54.27 in.; inside diameter $= 54.27 - 2.5 = 51.77$ in.; area of upper base $= 51.77^2 \times .7854 = 2{,}105$ sq. in., nearly.

Outside diameter of lower base $= \frac{190}{3.1416} = 60.48$ in., nearly; inside diameter $= 60.48 - 2.5 = 57.98$ in.; area of lower base $= 57.98^2 \times .7854 = 2{,}640$ sq. in., nearly.

Apply rule **67**, Art. **427**. Contents of the tank in cubic inches $= (2{,}105 + 2{,}640 + \sqrt{2{,}105 \times 2{,}640}) \times \frac{7 \times 12}{3} = (4{,}745 + 2{,}357) \times 28 = 7{,}102 \times 28 = 198{,}856$ cu. in. Hence, the number of gallons $= 198{,}856 \div 231 = 860.8$. Therefore, in round numbers, the tank will hold 861 gallons.

(210) (*a*) By rule **68**, Art. **429**, area of the surface $= 22.5^2 \times 3.1416 = 506.25 \times 3.1416 = 1{,}590.435$ sq. in. Ans.

(*b*) Using rule **69**, Art. **430**, the cubical contents = the cube of the diameter $\times .5236 = 11{,}390.625 \times .5236 = 5{,}964.1313$ cu. in. Ans.

(211) Having given the area of the surface, to find the volume we must first obtain the diameter. The process is just the reverse of finding the surface when the diameter is given. Hence, the diameter $= \sqrt{201.0624 \div 3.1416} = \sqrt{64} = 8$ in. By rule **69**, Art. **430**, volume $= 8^3 \times .5236 = 268.0832$ cu. in. Ans.

(212) (*a*) Given $OB = \frac{16}{2}$, or 8 inches, and $OA = \frac{13}{2}$, or $6\frac{1}{2}$ inches, to find the volume, area, and weight (see Fig. 12):

Radius of center circle equals $\frac{8 + 6.5}{2}$, or $7\frac{1}{4}$ inches.

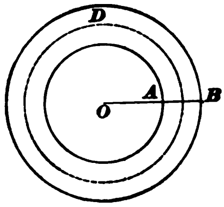

FIG. 12.

Length of center line $= 2 \times 3.1416 \times 7\frac{1}{4} = 45.5532$ inches.

The radius of the inner circle is $6\frac{1}{2}$ inches, and of the outer circle 8 inches; therefore, the diameter of the cross-section on the line AB is $1\frac{1}{2}$ inches. Then, according to rule **70**, Art. **431**, the area of the ring is $1\frac{1}{2} \times 3.1416 \times 45.553 = 214.665$ square inches. Ans.

Diameter of cross-section of ring $= 1\frac{1}{2}$ inches.

Area of cross-section of ring $= (1\frac{1}{2})^2 \times .7854 = 1.76715$ sq. in. Ans.

By rule **71**, Art. **432**, volume of ring $= 1.76715 \times 45.553 = 80.499$ cu. in. Ans.

(*b*) Weight of ring $= 80.499 \times .261 = 21$ lb. Ans.

(213) The volume of the ring equals the product of its cross-sectional area and its length on line D; therefore,

cross-sectional area $= \frac{144.349}{20.42} = 7.069$ sq. in.

The diameter $AB = \sqrt{\frac{7.069}{.7854}} = 3$ in.

Area of surface = circumference of circle, of which AB is diameter, $\times$ length $= 3.1416 \times 3 \times 20.42 = 192.45$ square inches. Ans.

ELEMENTARY ALGEBRA

AND

TRIGONOMETRIC FUNCTIONS.

(QUESTIONS 214–268.)

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(214) See Art. **531.**

(215) (*a*) $3x + 6 - 2x = 7x$. Transposing 6 to the second member, and $7x$ to the first member (Art. **575**),

$$3x - 2x - 7x = -6.$$

Combining like terms, $-6x = -6$;

whence, $x = 1$. Ans.

(*b*) $5x - (3x - 7) = 4x - (6x - 35)$.

Removing the parentheses (Art. **482**),

$$5x - 3x + 7 = 4x - 6x + 35.$$

Transposing 7 to the second member, and $4x$ and $-6x$ to the first member,

$$5x - 3x - 4x + 6x = 35 - 7. \quad \text{(Art. \textbf{575}.)}$$

Combining like terms, $4x = 28$;

whence, $x = 28 \div 4 = 7$. Ans.

(*c*) $(x + 5)^2 - (4 - x)^2 = 21x$.

Performing the operations indicated, the equation becomes

$$x^2 + 10x + 25 - 16 + 8x - x^2 = 21x.$$

Transposing, $x^2 - x^2 + 10x + 8x - 21x = 16 - 25$.

Combining like terms, $-3x = -9$.

Dividing by -3, $x = 3$.

(216)

$$a^3-a^2-2a-1)\,2a^6-4a^5-5a^4+3a^3+10a^2+7a+2\,(2a^3-2a^2-3a-2.$$

$$\begin{array}{rrrrrrr}
\underline{2a^6} & \underline{-\,2a^5} & \underline{-\,4a^4} & \underline{-\,2a^3} & & & \text{Ans.}\\
 & -\,2a^5 & -\,a^4 & +\,5a^3 & +\,10a^2 & & \\
 & \underline{-\,2a^5} & \underline{+\,2a^4} & \underline{+\,4a^3} & \underline{+\,2a^2} & & \\
 & & -\,3a^4 & +\,a^3 & +\,8a^2 & +\,7a & \\
 & & \underline{-\,3a^4} & \underline{+\,3a^3} & \underline{+\,6a^2} & \underline{+\,3a} & \\
 & & & -\,2a^3 & +\,2a^2 & +\,4a & +\,2\\
 & & & \underline{-\,2a^3} & \underline{+\,2a^2} & \underline{+\,4a} & \underline{+\,2}
\end{array}$$

(217) (*a*)

$$\begin{array}{l}
2+4a-5a^2-6a^3\\
\underline{7a^3\qquad\qquad\qquad\qquad}\\
14a^3+28a^4-35a^5-42a^6.\quad \text{Ans. (Art. } \mathbf{493.}\text{)}
\end{array}$$

(*b*)

$$\begin{array}{l}
4x^2-4y^2+6z^2\\
\underline{3x^2y\qquad\qquad\qquad\qquad}\\
12x^4y-12x^2y^3+18x^2yz^2.\quad \text{Ans.}
\end{array}$$

(*c*)

$$\begin{array}{l}
3b+5c-2d\\
\underline{6a\qquad\qquad\qquad\qquad}\\
18ab+30ac-12ad.\quad \text{Ans.}
\end{array}$$

(218) Let x = number of miles he traveled per hour.

Then, $\frac{48}{x}$ = time it took him.

$\frac{48}{x+4}$ = time it would take him if he traveled 4 miles more per hour.

In the latter case the time would have been 6 hours; hence the equation

$$\frac{48}{x+4}=6.$$

Clearing of fractions, $48 = 6\,(x+4)$.

Dividing by 6, $8 = x + 4$.

Transposing, $8 - 4 = x$.

$x = 4$. Ans.

(219) The square root of the fraction *a* plus *b* plus *c* over *n*, plus the square root of *a*, plus the fraction *b* plus *c* over *n*, plus the square root of the quantity *a* plus *b*, plus the fraction c over n, plus the parenthesis a plus b, times c, plus a plus bc.

(220) (a) Writing the work as follows, and canceling common factors in both numerator and denominator (Arts. **545** and **546**), we have

$$\frac{9m^2n^2}{8p^3q^3} \times \frac{5p^2q}{2xy} \times \frac{24x^2y^2}{90mn} =$$

$$\frac{9 \times 5 \times 24 \times m^2 \times n^2 \times p^2 \times q \times x^2 \times y^2}{8 \times 2 \times 90 \times m \times n \times p^3 \times q^3 \times x \times y} = \frac{3mnxy}{4pq^2}. \text{ Ans.}$$

(b) This problem may be written as follows, according to Art. **529**,

$$\frac{3ax+4}{1} \times \frac{a^2}{a(3ax+4)\,(3ax+4)}.$$

Canceling a and $(3ax + 4)$, we have $\frac{a}{3ax+4}$. Ans.

(221) See Arts. **602** and **603**.

Sin 17° 28′ = .30015.
Sin 17° 27′ = .29987.

30015 − 29987 = 28, difference for 1′.
$28 \times \frac{37}{60} = 17$, difference for 37″.
29987 + 17 = 30004.

Hence, locating the decimal point,
Sin 17° 27′ 37″ = .30004.

Cos 17° 27′ = .95398.
Cos 17° 28′ = .95389.
95398 − 95389 = 9, difference for 1′.
$9 \times \frac{37}{60} = 6$, difference for 37″.
95398 − 6 = 95392.

Hence, Cos 17° 27′ 37″ = .95392.

Tan 17° 28′ = .31466.
Tan 17° 27′ = .31434.

31466 − 31434 = 32, difference for 1′.
$32 \times \frac{37}{60} = 20$, difference for 37″.
31434 + 20 = 31454.

Hence, Tan 17° 27′ 37″ = .31454.

$$\left.\begin{aligned}\text{Sin } 17^\circ\, 27'\, 37'' &= .30004.\\ \text{Cos } 17^\circ\, 27'\, 37'' &= .95392.\\ \text{Tan } 17^\circ\, 27'\, 37'' &= .31454.\end{aligned}\right\}\text{ Ans.}$$

(222) Let $x =$ the capacity.
Then, $x - 42 =$ amount held at first.

$$7(x - 42) = x.$$
$$7x - 294 = x.$$
$$6x = 294.$$
$$x = 49 \text{ gallons.} \quad \text{Ans.}$$

(223) $\frac{c(a+b)+cd}{(a+b)c} = \frac{ac+bc+cd}{ac+bc}$. Canceling c, which is common to each term, we have $\frac{a+b+d}{a+b} = 1 + \frac{d}{a+b}$. Ans.

(224) The solution is exactly similar to that of example 221, preceding.

$$\begin{aligned}\text{Sin of } 63^\circ\, 4'\, 51.8'' &= .89165.\\ \text{Cos of } 63^\circ\, 4'\, 51.8'' &= .45274.\\ \text{Tan of } 63^\circ\, 4'\, 51.8'' &= 1.96949.\end{aligned}$$

(225) $(c^{-\frac{2}{5}})^{-\frac{1}{3}} = c^{\frac{2}{15}}$. Ans.

$$(m\sqrt{n^3})^{-\frac{1}{2}} = m^{-\frac{1}{2}}(n^{\frac{3}{2}})^{-\frac{1}{2}} = m^{-\frac{1}{2}}n^{-\frac{3}{4}} = \frac{1}{m^{\frac{1}{2}}n^{\frac{3}{4}}}. \quad \text{Ans.}$$

$$(cd^{-2})^{\frac{1}{a}} = c^{\frac{1}{a}}d^{-\frac{2}{a}}, \text{ or } \sqrt[a]{cd^{-2}}, \text{ or } \sqrt[a]{\frac{c}{d^2}}. \quad \text{Ans.} \quad (\text{Art. } \mathbf{565.})$$

(226) $\frac{3+2x}{2-x} - \frac{2-3x}{2+x} + \frac{16x-x^2}{x^2-4}$. If the denominator of the third fraction were written $4 - x^2$, instead of $x^2 - 4$, the common denominator would then be $4 - x^2$.

By Art. **531**, $\frac{16x-x^2}{x^2-4}$ becomes $-\frac{16x-x^2}{-x^2+4} = -\frac{16x-x^2}{4-x^2}$.

Hence, $\frac{3+2x}{2-x} - \frac{2-3x}{2+x} - \frac{16x-x^2}{4-x^2}$, when reduced to a common denominator, becomes

$$\frac{(3+2x)(2+x)-(2-3x)(2-x)-(16x-x^2)}{4-x^2}=$$

$$\frac{(6+7x+2x^2)-(4-8x+3x^2)-(16x-x^2)}{4-x^2}.$$

Removing the parentheses (Art. **482**), we have

$$\frac{6+7x+2x^2-4+8x-3x^2-16x+x^2}{4-x^2}.$$

Combining like terms in the numerator, we have

$$\frac{2-x}{4-x^2}.$$

Factoring the denominator (Art. **523**), we have

$$\frac{2-x}{(2+x)(2-x)}.$$

Canceling the common factor $(2-x)$, the result is equal to

$$\frac{1}{2+x}, \text{ or } \frac{1}{x+2}. \quad \text{Ans.} \quad (\text{Art. } \mathbf{456.})$$

(227) $$\frac{a^2+c^2+ac}{a^2+b^2-c^2-2ab}\times\frac{a^2+c^2-b^2-2ac}{a^3c+a^2c^2+ac^3}.$$

Arranging the terms, we have

$$\frac{a^2+ac+c^2}{a^2-2ab+b^2-c^2}\times\frac{a^2-2ac+c^2-b^2}{a^3c+a^2c^2+ac^3},$$

which, by the use of parentheses, becomes

$$\frac{a^2+ac+c^2}{(a^2-2ab+b^2)-c^2}\times\frac{(a^2-2ac+c^2)-b^2}{a^3c+a^2c^2+ac^3}.$$

By Art. **518,** we know that $a^2-2ab+b^2$ and $a^2-2ac+c^2$ are perfect squares, and may be written $(a-b)^2$ and $(a-c)^2$.

Factoring $a^3c+a^2c^2+ac^3$ by Case I, Art. **516,** we have

$$\frac{a^2+ac+c^2}{(a-b)^2-c^2}\times\frac{(a-c)^2-b^2}{ac(a^2+ac+c^2)}=$$

$$\frac{a^2+ac+c^2}{(a-b-c)(a-b+c)}\times\frac{(a-c-b)(a-c+b)}{ac(a^2+ac+c^2)}.$$

(Art. **523.**)

Canceling common factors and multiplying, we have

$$\frac{a-c+b}{(a-b+c)ac}, \text{ or } \frac{a+b-c}{ac(a-b+c)}. \quad \text{Ans.}$$

(228) If none of the terms is similar, the subtraction of one expression from another may be represented only, by connecting the subtrahend with the minuend by means of the sign $-$. Thus, if it is required to subtract $5a^3b - 7a^2b^2 + 5ab^3$ from $a^4 - b^4$, the result will be represented by $a^4 - b^4 - (5a^3b - 7a^2b^2 + 5ab^3)$, which, on removing the parenthesis (Art. **482**), becomes $a^4 - b^4 - 5a^3b + 7a^2b^2 - 5ab^3$. From this result subtract $3a^4 - 4a^3b + 6a^2b^2 + 5ab^3 - 3b^4$.

$$\begin{array}{rl} a^4 - b^4 - 5a^3b + 7a^2b^2 - 5ab^3 & \textit{minuend.} \\ -3a^4 + 3b^4 + 4a^3b - 6a^2b^2 - 5ab^3 & \textit{subtrahend}\text{, with signs changed.} \\ \hline -2a^4 + 2b^4 - a^3b + a^2b^2 - 10ab^3 & \textit{remainder.} \qquad \text{(Art. } \mathbf{479.}\text{)} \end{array}$$

Or, $-2a^4 - a^3b + a^2b^2 - 10ab^3 + 2b^4$. Ans. arranged according to the decreasing powers of a.

(229) $.27038 = \sin 15° \, 41' \, 12.9''$. (Art. **606.**)
$.27038 = \cos 74° \, 18' \, 47.1''$.
$2.27038 = \tan 66° \, 13' \, 43.2''$.

(230) (*a*) If the work done *on* a piston by the confined gases be considered positive, then the work done *by* the piston in pushing the gas out of the exhaust-port may be considered negative.

(*b*) While in arithmetic we can add and subtract only positive quantities, in algebra we can perform these operations on both positive and negative quantities.

(231) (*a*) The value of a^0 is 1. (Art. **503.**)

(*b*) $\frac{a^0}{a^{-1}} = a$. Ans. (Art. **565.**)

(232) $-\frac{c-(a-b)}{c+(a+b)} = \frac{(a-b)-c}{c+(a+b)}$. Ans. (Art. **531.**)

(233) (*a*) By Art. **530,** the reciprocal of $\frac{24}{49} = 1 \div \frac{24}{49} = \times \frac{49}{24} = \frac{49}{24}$. Ans.

(*b*) Since, by Art. **530**, a number may be found from its reciprocal by dividing 1 by the reciprocal, the number $= 1 \div 700 = .0014\frac{2}{7}$. Ans.

(234) (*a*)

$$\begin{array}{r} 3a - 2b + 3c \\ \underline{2a - 8b - c} \end{array} \quad \text{becomes} \quad \begin{array}{r} 3a - 2b + 3c \\ \underline{-2a + 8b + c} \\ a + 6b + 4c \end{array}$$

when the signs of the subtrahend are changed. Now, adding each term (with its sign changed) in the subtrahend to its corresponding term in the minuend, we have $(-2a) + (3a) = a$; $(+8b) + (-2b) = +6b$; $(+c) + (3c) = +4c$. Hence, $a + 6b + 4c$ equals the difference. Ans.

(*b*)

$$\begin{array}{l} 2x^3 - 3x^2y + 2xy^3 \\ \underline{x^3 \qquad\qquad\qquad\qquad + y^3 - xy^2} \end{array} \text{ becomes } \begin{array}{r} 2x^3 - 3x^2y + 2xy^3 \\ \underline{-x^3 \qquad\qquad\qquad\qquad - y^3 + xy^2} \\ x^3 - 3x^2y + 2xy^3 - y^3 + xy^2 \end{array}$$

when the signs of the subtrahend are changed. Adding each term in the subtrahend (with its sign changed) to its corresponding term in the minuend, we have $x^3 - 3x^2y + 2xy^3 - y^3 + xy^2$, which, arranged according to the decreasing powers of x, equals $x^3 - 3x^2y + xy^2 + 2xy^3 - y^3$. Ans.

(*c*)

$$\begin{array}{r} 14a + 4b - 6c - 3d \\ \underline{11a - 2b + 4c - 4d} \end{array}$$

On changing the sign of each term in the subtrahend, the problem becomes

$$\begin{array}{r} 14a + 4b - 6c - 3d \\ \underline{-11a + 2b - 4c + 4d} \\ 3a + 6b - 10c + d \end{array}$$

Adding each term of the subtrahend (with the sign changed) to its corresponding term in the minuend, the difference, or result, is $3a + 6b - 10c + d$. Ans.

(235) The numerical values of the following, when $a = 16$, $b = 10$, and $x = 5$, are:

(*a*) $(ab^2x + 2abx)\,4a = (16 \times 10^2 \times 5 + 2 \times 16 \times 10 \times 5) \times 4 \times 16$. It must be remembered that when no sign is expressed between symbols or quantities, the sign of multiplication is understood.

$(16 \times 100 \times 5 + 2 \times 16 \times 10 \times 5) \times 64 = (8,000 + 1,600) \times 64 = 9,600 \times 64 = 614,400$. Ans.

(*b*)
$$2\sqrt{4a} - \frac{2bx}{a-b} + \frac{b-x}{x} =$$
$$2\sqrt{64} - \frac{2 \times 10 \times 5}{16-10} + \frac{10-5}{5} =$$
$$16 - \tfrac{100}{6} + 1 = \frac{96 - 100 + 6}{6} = \tfrac{2}{6} = \tfrac{1}{3}.\ \text{Ans.}$$

(*c*) $(b - \sqrt{a})\,(x^3 - b^2)\,(a^2 - b^2) = (10 - \sqrt{16}) \times (5^3 - 10^2) \times (16^2 - 10^2) = (10 - 4)\,(125 - 100)\,(256 - 100) = 6 \times 25 \times 156 = 23,400$. Ans.

(236) (*a*)
$$\begin{array}{r} 4xyz \\ -3xyz \\ -5xyz \\ 6xyz \\ -9xyz \\ 3xyz \\ \hline -4xyz \end{array} \quad \text{Ans.}$$

The sum of the coefficients of the positive terms we find to be $+13$, since $(+3) + (+6) + (+4) = (+13)$.

When no sign is given before a quantity, the $+$ sign must always be understood. The sum of the coefficients of the negative terms we find to be -17, since $(-9) + (-5) + (-3) = (-17)$. Subtracting the *lesser* sum from the *greater* and prefixing the sign of the greater sum $(-)$ (Art. **471,** Rule II), we have $(+13) + (-17) = -4$. Since the terms are all alike, we have only to annex the common symbols xyz to -4, thereby obtaining $-4xyz$ for the result, or sum.

(*b*)
$$\begin{array}{rrr} 3a^2 + & 2ab + & 4b^2 \\ 5a^2 - & 8ab + & b^2 \\ -a^2 + & 5ab - & b^2 \\ 18a^2 - & 20ab - & 19b^2 \\ 14a^2 - & 3ab + & 20b^2 \\ \hline 39a^2 - & 24ab + & 5b^2. \end{array} \quad \text{Ans.}$$

When adding polynomials, always place like terms under each other. (Art. **478.**)

The coefficient of a^2 in the result will be 39, since $(+14) + (+18) + (-1) +$

$(+5) + (+3) = 39$. When the coefficient of a term is not written, 1 is always understood to be its coefficient. (Art. **442.**) The coefficient of ab will be -24, since $(-3) + (-20) + (+5) + (-8) + (+2) = -24$. The coefficient of b^2 will be $(+20) + (-19) + (-1) + (+1) + (+4) = +5$. Hence, the result, or sum, is $39a^2 - 24ab + 5b^2$.

(*c*)

$$\begin{array}{llllll} 4mn & + 3ab & - 4c & & & \\ +2mn & - 4ab & & + 3x & + 3m^2 & - 4p \\ \hline 6mn & - ab & - 4c & + 3x & + 3m^2 & - 4p. \end{array} \quad \text{Ans.}$$

(237) (*a*) See Art. **445.**

(*b*) In multiplication, coefficients are multiplied, and exponents are added. In division, the coefficients of the dividend are divided by those of the divisor, and the exponents of the divisor are subtracted from those of the dividend. See rules of multiplication and division.

(*c*) See Art. **487.**

(238) The side $BC = \sqrt{\overline{AB}^2 - \overline{AC}^2}$, or $BC = \sqrt{17.69^2 - 9.75^2} = \sqrt{217.8736} = 14$ ft. 9 in. To find the angle BAC, we have

$$\cos BAC = \frac{AC}{AB}, \text{ or } \cos BAC = \frac{9.75}{17.69} = .55115. \quad (\text{Art. } \mathbf{595.})$$

$$.55115 = \cos 56^\circ\, 33'\, 15''.$$

Angle $ABC = 90^\circ -$ angle BAC, or $90^\circ - 56^\circ\, 33'\, 15'' = 33^\circ\, 26'\, 45''$.

$$\left.\begin{array}{l} \text{Side } BC = 14 \text{ ft. } 9 \text{ in.} \\ \text{Angle } BAC = 56^\circ\, 33'\, 15''. \\ \text{Angle } ABC = 33^\circ\, 26'\, 45''. \end{array}\right\} \text{Ans.}$$

(239) (*a*) $$\frac{9x + 20}{36} = \frac{4(x - 3)}{5x - 4} + \frac{x}{4}.$$

When the denominators contain both simple and compound expressions, it is best to remove the simple expressions first, and then remove each compound expression in order. Then, after each multiplication, the result should be reduced to the simplest form.

Multiplying both sides by 36,

$$\mathbf{9x + 20 = \frac{144\,(x - 3)}{5x - 4} + 9x,}$$

or

$$\mathbf{\frac{144x - 432}{5x - 4} = 20.}$$

Clearing of fractions,

$$\mathbf{144x - 432 = 100x - 80.}$$

Transposing and combining,

$$44x = 352;$$

whence, $x = 8.$ Ans.

(*b*) $ax - \frac{3a - bx}{2} = \frac{1}{2}$ becomes, when cleared of fractions,

$$2ax - 3a + bx = 1.$$

Transposing and uniting terms,

$$2ax + bx = 3a + 1.$$

Factoring, $(2a + b)x = 3a + 1;$

whence, $x = \frac{3a + 1}{2a + b}.$ Ans.

(*c*) $am - b - \frac{ax}{b} + \frac{x}{m} = 0$ becomes, when cleared of fractions,

$$abm^2 - b^2m - amx + bx = 0.$$

Transposing, $bx - amx = b^2m - abm^2.$

Factoring, $(b - am)x = bm(b - am);$

whence, $x = \frac{bm(b - am)}{(b - am)} = bm.$ Ans.

(240) (*a*) *a* square *x* square, plus two *a* cube *b* fifth, minus the parenthesis *a* plus *b*.

(*b*) The cube root of *x*, plus *y* times the two-thirds power of the parenthesis *a* minus *n* square.

(*c*) The parenthesis *m* plus *n*, times the square of the parenthesis *m* minus *n*, times the parenthesis *m* minus the fraction *n* over two.

(241) (*a*) $16a^2b^3$; $a^4 + 4ab$; $4a^2 - 16a^3b + 5a^6 + 7ax$.

(*b*) Since the terms are not alike, we can only indicate the sum, connecting the terms by their proper signs. (Art. **476.**)

(*c*) Multiplication: $4ac^2d$ means $4 \times a \times c^2 \times d$. (Art. **441.**)

(242) (*a*) $45x^7y^{10} - 90x^5y^{11} - 360x^4y^8 =$
$45x^4y^7(x^3y^3 - 2x - 8y)$. Ans. (Art. **516.**)

(*b*) $a^2b^2 + 2abcd + c^2d^2 = (ab + cd)^2$. Ans. (Art. **519.**)

(*c*) $(a + b)^2 - (c - d)^2 = (a + b + c - d)(a + b - c + d)$. Ans. (Art. **523.**)

(243) (*a*) On removing the vinculum, we have

$$2a - \{3b + [4c - 4a - (2a + 2b)] + [3a - b - c]\}.$$

(Art. **482.**)

Removing the parenthesis,

$$2a - \{3b + [4c - 4a - 2a - 2b] + [3a - b - c]\}.$$

Removing the brackets,

$$2a - \{3b + 4c - 4a - 2a - 2b + 3a - b - c\}.$$

Removing the brace,

$$2a - 3b - 4c + 4a + 2a + 2b - 3a + b + c.$$

Combining like terms, the result is $5a - 3c$. Ans.

(*b*) Removing the parenthesis, we have

$$7a - \{3a - [2a - 5a + 4a]\}.$$

Removing the brackets,

$$7a - \{3a - 2a + 5a - 4a\}.$$

Removing the brace,

$$7a - 3a + 2a - 5a + 4a.$$

Combining terms, the result is $5a$. Ans.

(*c*) Removing the parenthesis, we have

$$a - \{2b + [3c - 3a - a - b] + [2a - b - c]\}.$$

Removing the brackets,

$$a - \{2b + 3c - 3a - a - b + 2a - b - c\}.$$

Removing the brace,

$$a - 2b - 3c + 3a + a + b - 2a + b + c.$$

Combining like terms, the result is $3a - 2c$. Ans.

(244) The hypotenuse $AB = \sqrt{\overline{AC}^2 + \overline{BC}^2}$, or

$$AB = \sqrt{17.5^2 + 21.3^2} = \sqrt{759.94} = 27.57, \text{ nearly. Ans.}$$

$$\text{Tan } A = \frac{AC}{BC} = \frac{17.5}{21.3} = .82160. \quad (\text{Art. } \mathbf{596.})$$

$$.82160 = \tan 39^\circ\, 24'\, 23''.$$

Angle $A = 39^\circ\, 24'\, 23''$. Ans.
Angle $B = 90^\circ -$ angle $A = 90^\circ - 39^\circ\, 24'\, 23'' = 50^\circ\, 35'\, 37''$. Ans.

(245)
$$t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2}{W_1 s_1 + W_2 s_2}.$$

In order to transform this formula so that t_2 may stand alone in the first member, we must first clear of fractions. Performing this operation, we have

$$t W_1 s_1 + t W_2 s_2 = W_1 s_1 t_1 + W_2 s_2 t_2.$$

Transposing, we have

$$- W_2 s_2 t_2 = W_1 s_1 t_1 - t W_1 s_1 - t W_2 s_2.$$

Factoring, we have

$$- W_2 s_2 t_2 = W_1 s_1 t_1 - (W_1 s_1 + W_2 s_2) t;$$

whence,
$$t_2 = \frac{(W_1 s_1 + W_2 s_2)\, t - W_1 s_1 t_1}{W_2 s_2}. \quad \text{Ans.}$$

(246) Let $x =$ the part of the work which they all can do in 1 day when working together.

Then, since $\frac{1}{7\frac{1}{2}} = \frac{2}{15}$, $\quad \frac{1}{5} + \frac{1}{6} + \frac{2}{15} = x$;

or, clearing of fractions and adding,

$$15 = 30x, \text{ and } x = \tfrac{1}{2}.$$

Since they can do $\frac{1}{2}$ the work in 1 day, they can do all the work in 2 days. Ans.

(247) (*a*)
$$\frac{\dfrac{1}{1-x} - \dfrac{1}{1+x}}{\dfrac{1}{1-x} + \dfrac{1}{1+x}} = \frac{\dfrac{1+x-1+x}{1-x^2}}{\dfrac{1+x+1-x}{1-x^2}} =$$

$$\frac{2x}{1-x^2} \div \frac{2}{1-x^2} = \frac{2x}{1-x^2} \times \frac{1-x^2}{2} = x. \quad \text{Ans. (See Art. } \mathbf{559.})$$

(*b*) $$\frac{\frac{a^2}{b^3}+\frac{1}{a}}{\frac{a}{b^2}-\frac{a-b}{ab}}=\frac{\frac{a^3+b^3}{ab^3}}{\frac{a^2b-b^2(a-b)}{ab^3}}=\frac{\frac{a^3+b^3}{ab^3}}{\frac{a^2b-ab^2+b^3}{ab^3}}=$$

$$\frac{a^3+b^3}{ab^3}\div\frac{a^2b-ab^2+b^3}{ab^3}=\frac{a^3+b^3}{ab^3}\times\frac{ab^3}{b(a^2-ab+b^2)}=$$

$$\frac{a+b}{b}.\quad \text{Ans.}$$

(*c*) $$\frac{1}{x+\frac{1}{1+\frac{x+1}{3-x}}}=\frac{1}{x+\frac{3-x}{4}}=\frac{4}{3x+3}.\quad \text{Ans.}$$

(Art. **559.**)

(248) (*a*) $6a^4b^4+a^3b^2-7a^2b^3+2abc+3$.

(*b*) $3+2abc+a^3b^2-7a^2b^3+6a^4b^4$.

(*c*) $1+ax+a^2+2a^3$. Written like this, the a in the second term is understood as having 1 for an exponent; hence, if we represent the first term by a^0, in value it will be equal to 1, since $a^0=1$. Therefore, 1 should be written as the first term when arranged according to the increasing powers of a.

(249) (*a*) According to Art. **563,** $x^{\frac{3}{4}}$ expressed radically is $\sqrt[4]{x^3}$;

$3x^{\frac{1}{4}}y^{-\frac{3}{4}}$ expressed radically is $3\sqrt{xy^{-3}}$;

$3x^{\frac{1}{6}}y^{-\frac{5}{6}}z^{\frac{1}{3}}=3\sqrt[6]{xy^{-5}z^2}$, since $z^{\frac{1}{3}}=z^{\frac{2}{6}}$. Ans.

(*b*) (See Art. **565.**) $$a^{-1}b^{\frac{1}{2}}+\frac{c^{-2}}{a+b}+(m-n)^{-1}-\frac{a^2b^{-2}c}{c^{-3}}=$$

$$\frac{b^{\frac{1}{2}}}{a}+\frac{1}{c^2(a+b)}+\frac{1}{m-n}-\frac{a^2c^4}{b^2}.\quad \text{Ans.}$$

(*c*) $\sqrt[7]{x^6}=x^{\frac{6}{7}}$. Ans. $\sqrt[3]{x^{-4}}=x^{-\frac{4}{3}}$. Ans.

$(\sqrt[4]{b^5x^2})^3=(b^{\frac{5}{4}}x^{\frac{1}{2}})^3=b^{\frac{15}{4}}x^{\frac{3}{2}}$. Ans.

(250) (*a*) $$\frac{2ax+x^2}{a^3-x^3}\div\frac{x}{a-x}=\frac{x(2a+x)}{a^3-x^3}\times\frac{a-x}{x}.$$

(Art. **549.**)

Canceling common factors, the result is $\frac{2a+x}{a^2+ax+x^2}$. Ans.

$$\begin{array}{l}
a-x\,)\,a^3-x^3\,(\,a^2+ax+x^2 \\
\quad\ \ \underline{a^3-a^2x} \\
\qquad\quad a^2x-x^3 \\
\qquad\quad \underline{a^2x-ax^2} \\
\qquad\qquad\quad ax^2-x^3 \\
\qquad\qquad\quad \underline{ax^2-x^3}
\end{array}$$

(*b*) Inverting the divisor and factoring, we have

$$\frac{3n(2m^2n-1)}{(2m^2n-1)\,(2m^2n-1)} \times \frac{(2m^2n+1)\,(2m^2n-1)}{3n}.$$

Canceling common factors, the result is $2m^2n+1$. Ans.

(*c*) $9+\frac{5y^2}{x^2-y^2} \div \left(3+\frac{5y}{x-y}\right)$ simplified gives

$\frac{9x^2-4y^2}{x^2-y^2} \div \frac{3x+2y}{x-y}$.

Inverting the divisor, we have $\frac{9x^2-4y^2}{x^2-y^2} \times \frac{x-y}{3x+2y}$. Canceling common factors, the result is $\frac{3x-2y}{x+y}$. Ans.

(251) (*a*)
$$\begin{array}{rrrrr}
2x^3 & +\,2x^2 & +\,2x & -\,2 & \\
x & -\,1 & & & \\
\hline
2x^4+2x^3 & +\,2x^2 & -\,2x & & \\
-\,2x^3 & -\,2x^2 & -\,2x & +\,2 & \\
\hline
2x^4 & & -\,4x & +\,2. & \text{Ans.}
\end{array}$$

(*b*)
$$\begin{array}{rrrrr}
x^2 & -\,4ax & +\,c & & \\
2x & +\,a & & & \\
\hline
2x^3 & -\,8ax^2 & +\,2cx & & \\
 & ax^2 & & -\,4a^2x & +\,ac \\
\hline
2x^3 & -\,7ax^2 & +\,2cx & -\,4a^2x & +\,ac. \quad \text{Ans.}
\end{array}$$

(*c*)
$$\begin{array}{rrrrr}
-\,a^3 & +\,3a^2b & -\,2b^3 & & \\
5a^2 & +\,9ab & & & \\
\hline
-\,5a^5 & +\,15a^4b & -\,10a^2b^3 & & \\
 & -\,9a^4b & & +\,27a^3b^2 & -\,18ab^4 \\
\hline
-\,5a^5 & +\,6a^4b & -\,10a^2b^3 & +\,27a^3b^2 & -\,18ab^4
\end{array}$$

Arranging the terms according to the decreasing powers of a, we have

$$-5a^5+6a^4b+27a^3b^2-10a^2b^3-18ab^4. \quad \text{Ans.}$$

(252) (a) $\dfrac{x}{x-y}+\dfrac{x-y}{y-x}$. If the denominator of the second fraction were written $x-y$, instead of $y-x$, then $x-y$ would be the common denominator.

By Art. **531**, the signs of the denominator and the sign before the fraction $\dfrac{x-y}{y-x}$ may be changed, giving $-\dfrac{x-y}{x-y}$. We now have

$$\frac{x}{x-y}-\frac{x-y}{x-y}=\frac{x-x+y}{x-y}=\frac{y}{x-y}. \quad \text{Ans.}$$

(b) $\dfrac{x^2}{x^2-1}+\dfrac{x}{x+1}-\dfrac{x}{1-x}$. If we write the denominator of the third fraction $x-1$ instead of $1-x$, x^2-1 will then be the common denominator.

By Art. **531**, the signs of the denominator and the sign before the fraction may be changed, thereby giving $\dfrac{x}{x-1}$. We now have

$$\frac{x^2}{x^2-1}+\frac{x}{x+1}+\frac{x}{x-1}=\frac{x^2+x(x-1)+x(x+1)}{x^2-1}=$$

$$\frac{x^2+x^2-x+x^2+x}{x^2-1}=\frac{3x^2}{x^2-1}. \quad \text{Ans.}$$

(c) $$\frac{3a-4b}{7}-\frac{2a-b+c}{3}+\frac{13a-4c}{12}$$

when reduced to a common denominator =

$$\frac{12(3a-4b)-28(2a-b+c)+7(13a-4c)}{84}.$$

Expanding the terms and removing the parentheses, we have

$$\frac{36a-48b-56a+28b-28c+91a-28c}{84}.$$

Combining like terms in the numerator, we have as the result,

$$\frac{71a - 20b - 56c}{84}. \quad \text{Ans.}$$

(253) (*a*) Factoring each expression (Art. **519**), we have

$$9x^4 + 12x^2y^2 + 4y^4 = (3x^2 + 2y^2)(3x^2 + 2y^2) = (3x^2 + 2y^2)^2. \quad \text{Ans.}$$

(*b*) $49a^4 - 154a^2b^2 + 121b^4 = (7a^2 - 11b^2)(7a^2 - 11b^2) = (7a^2 - 11b^2)^2$. Ans.

(*c*) $64x^2y^2 + 64xy + 16 = 16(2xy + 1)^2$. Ans.

(254) (*a*) Arrange the dividend according to the decreasing powers of x and divide. Thus,

$$\begin{array}{l}
3x - 1\,)\,9x^3 + 3x^2 + x - 1\,(\,3x^2 + 2x + 1. \quad \text{Ans.}\\
\qquad\quad \underline{9x^3 - 3x^2}\\
\qquad\qquad\quad 6x^2 + x\\
\qquad\qquad\quad \underline{6x^2 - 2x}\\
\qquad\qquad\qquad\quad 3x - 1\\
\qquad\qquad\qquad\quad \underline{3x - 1}
\end{array}$$

(*b*)

$$\begin{array}{l}
a - b\,)\,a^3 - 2ab^2 + b^3\,(\,a^2 + ab - b^2. \quad \text{Ans.}\\
\qquad\quad \underline{a^3 - a^2b}\\
\qquad\qquad\quad a^2b - 2ab^2\\
\qquad\qquad\quad \underline{a^2b - ab^2}\\
\qquad\qquad\qquad - ab^2 + b^3\\
\qquad\qquad\qquad \underline{- ab^2 + b^3}
\end{array}$$

(*c*) Arranging the terms of the dividend according to the decreasing powers of x, we have

$$\begin{array}{l}
7x - 3\,)\,7x^3 - 24x^2 + 58x - 21\,(\,x^2 - 3x + 7. \quad \text{Ans.}\\
\qquad\quad \underline{7x^3 - 3x^2}\\
\qquad\quad - 21x^2 + 58x\\
\qquad\quad \underline{- 21x^2 + 9x}\\
\qquad\qquad\qquad 49x - 21\\
\qquad\qquad\qquad \underline{49x - 21}
\end{array}$$

(255) See Arts. **435** and **436.**

(256) (*a*)

$$1 + 2x - \frac{4x-4}{5x} = \frac{5x + 10x^2 - 4x + 4}{5x} = \frac{10x^2 + x + 4}{5x}. \text{ Ans.}$$

(Art. **555.**)

(*b*) $\frac{3x^2 + 2x + 1}{x+4} = 3x - 10 + \frac{41}{x+4}$. Ans. (Art. **554.**)

$$\begin{array}{r}
x + 4\,)\,3x^2 + \;\; 2x + 1\,(\,3x - 10 + \frac{41}{x+4} \\
\underline{3x^2 + 12x} \qquad\qquad\qquad\quad \\
-\,10x + \;\; 1 \qquad\qquad\qquad\quad \\
\underline{-\,10x - 40} \qquad\qquad\qquad\quad \\
41 \qquad\qquad\qquad\quad
\end{array}$$

(257) The angle $B = 90° -$ angle $A = 90° - 65° 13' 29'' =$ $24° 46' 31''$. Ans.

The side $B\,C = A\,B \times \sin A = 5.5$ yd. $\times \sin 65° 13' 29'' =$ 5.5 yd. $\times .90796 = 4.9938$ yd. (Art. **609.**) 4.9938 yd. $=$ 14 ft. 11¾ in. Ans.

Side $A\,C = A\,B \times \cos A = 5.5$ yd. $\times .41906 = 2.3048$ yd. $=$ 6 ft. 11 in., nearly. Ans.

(258) (*a*) $(x^4 - 1) \div (x^2 + 1) = (x^2 - 1)(x^2 + 1) \div$ $(x^2 + 1) = x^2 - 1$. Ans. (Art. **523.**)

(*b*) $x^4 - 2x^2y^2 + y^4 = (x^2 - y^2)^2$. (Art. **519.**) $(x^2 - y^2) =$ $(x + y)(x - y)$. (Art. **523.**)

Then $(x^2 - y^2)^2 = (x^2 - y^2)(x + y)(x - y)$. Dividing this latter quantity by $(x - y)$ we have $(x^2 - y^2)(x + y)$. Ans.

Note that $x - y$ is a factor of $(x^2 - y^2)^2$ and hence of $x^4 - 2x^2y^2 + y^4$.

(259) (*a*) $\frac{10x + 3}{3} - \frac{6x - 7}{2} = 10(x - 1)$.

Reducing the last member to a simpler form, the equation becomes

$$\frac{10x + 3}{3} - \frac{6x - 7}{2} = 10x - 10.$$

Clearing of fractions by multiplying each term of both members by 6, the common denominator, and changing the sign of each term of the numerator of the second fraction, since it is preceded by the minus sign, we have

$$20x + 6 - 18x + 21 = 60x - 60.$$

Transposing terms, $20x - 18x - 60x = -60 - 21 - 6.$

Combining like terms, $-58x = -87.$

Changing signs, $58x = 87;$

hence, $x = \frac{87}{58} = 1\frac{1}{2}.$ Ans.

(*b*) $(a^2 + x)^2 = x^2 + 4a^2 + a^4.$

Performing the operation indicated in the first member, the equation becomes

$$a^4 + 2a^2x + x^2 = x^2 + 4a^2 + a^4.$$

Canceling a^4 and x^2 (Art. **576**),

$$2a^2x = 4a^2.$$

Dividing by $2a^2$, $x = 2.$ Ans.

(*c*) $\frac{x-1}{x-2} - \frac{x+1}{x+2} = \frac{3}{x^2-4}.$

Clearing of fractions, the equation becomes

$$(x-1)(x+2) - (x+1)(x-2) = 3.$$

Expanding, $x^2 + x - 2 - x^2 + x + 2 = 3.$

Uniting terms, $2x = 3.$

$x = \frac{3}{2} = 1\frac{1}{2}.$ Ans.

(260)

$$\begin{array}{r} 159^\circ\ 27'\ 34.6'' \\ 25^\circ\ 16'\ 8.7'' \\ 3^\circ\ 48'\ 53.\ '' \\ \hline 188^\circ\ 32'\ 36.3'' \end{array}$$

(261) (*a*) $x + y + z - (x - y) - (y + z) - (-y)$ becomes $x + y + z - x + y - y - z + y$ on the removal of the parentheses. (Art. **482.**)

Combining like terms, $x - x + y + y - y + y + z - z = 2y.$ Ans.

(*b*) $(2x - y + 4z) + (-x - y - 4z) - (3x - 2y - z)$ becomes $2x - y + 4z - x - y - 4z - 3x + 2y + z$ on the removal of the parentheses. (Arts. **482** and **483.**) Combining like terms,

$$2x - x - 3x - y - y + 2y + 4z - 4z + z = z - 2x. \quad \text{Ans.}$$

(*c*) $a - [2a + (3a - 4a)] - 5a - \{6a - [(7a + 8a) - 9a]\}$.

In this expression we find aggregation marks of different shapes, thus, [, (, and {. In such cases look for the corresponding part (whatever may intervene), and all that is included between the two parts of each aggregation mark must be treated as directed by the sign before it (Arts. **482** and **483**), no attention being given to any of the other aggregation marks. It is always best to begin with the *innermost* pair, and remove each pair of aggregation marks in order. First removing the parentheses, we have

$$a - [2a + 3a - 4a] - 5a - \{6a - [7a + 8a - 9a]\}.$$

Removing the brackets, we have

$$a - 2a - 3a + 4a - 5a - \{6a - 7a - 8a + 9a\}.$$

Removing the brace, we have

$$a - 2a - 3a + 4a - 5a - 6a + 7a + 8a - 9a.$$

Combining like terms, the result is $-5a$. Ans.

(262) (*a*) $-7my\,)\,\underline{35m^3y + 28m^2y^2 - 14my^3}$

$-5m^2 - 4my + 2y^2$. Ans.

(Art. **506.**)

(*b*) $a^4\,)\,\underline{4a^4 - 3a^5b - a^6b^2}$

$4 - 3ab - a^2b^2$. Ans.

(*c*) $4x^2\,)\,\underline{4x^3 - 8x^5 + 12x^7 - 16x^9}$

$x - 2x^3 + 3x^5 - 4x^7$. Ans.

(263) Let $x =$ the length of the post.

Then, $\frac{x}{5} =$ the part in the earth.

$\frac{3x}{7} =$ the part in the water.

From the conditions of the problem, we have therefore the following statement:

$$\frac{x}{5} + \frac{3x}{7} + 13 = x;$$

from which $7x + 15x + 455 = 35x;$

$$-13x = -455;$$

and $x = 35$ feet. Ans.

(264) Let $x =$ the whole quantity.

Then, $\frac{2x}{3} + 10 =$ the quantity of niter.

$\frac{x}{6} - 4\frac{1}{2} =$ the quantity of sulphur.

$\frac{1}{7}\left(\frac{2x}{3} + 10\right) - 2 =$ the quantity of charcoal.

Hence, $$x = \frac{2x}{3} + 10 + \frac{x}{6} - 4\frac{1}{2} + \frac{1}{7}\left(\frac{2x}{3} + 10\right) - 2.$$

Clearing of fractions and expanding terms,

$$42x = 28x + 420 + 7x - 189 + 4x + 60 - 84.$$

Transposing,

$$42x - 28x - 7x - 4x = 420 - 189 + 60 - 84.$$

$$3x = 207.$$

$x = 69$ lb., the quantity of gunpowder. Ans.

$\frac{2x}{3} + 10 = \frac{2 \times 69}{3} + 10 = 56$ lb., the quantity of niter. Ans.

$\frac{x}{6} - 4\frac{1}{2} = \frac{69}{6} - 4\frac{1}{2} = 7$ lb., the quantity of sulphur. Ans.

56 lb. × $\frac{1}{7}$ − 2 lb. = 6 lb., the quantity of charcoal. Ans.

(265) (*a*) See Art. **568.** The cube root of -125 is -5. Dividing each of the exponents of the literal part by 3, the index of the root, the cube root of $x^3y^6z^9$ is $x^{\frac{3}{3}}y^{\frac{6}{3}}z^{\frac{9}{3}} = xy^2z^3$; hence, $\sqrt[3]{-125x^3y^6z^9} = -5xy^2z^3$. Ans.

(*b*) and (*c*) Proceed exactly as in (*a*). $\sqrt[4]{10,000} = \pm 10$ and $\sqrt[5]{243} = 3$; $\sqrt[4]{a^{16}b^{20}c^8} = a^{\frac{16}{4}}b^{\frac{20}{4}}c^{\frac{8}{4}} = a^4b^5c^2$, and $\sqrt[5]{m^{15}n^{20}} = m^3n^4$.

Hence, $\sqrt[4]{10,000a^{16}b^{20}c^8} = \pm 10a^4b^5c^2$. Ans.

$$\sqrt[5]{243m^{15}n^{20}} = 3m^3n^4. \quad \text{Ans.}$$

(*d*) Dividing the exponent of each letter in both numerator and denominator by 5, the index of the root,

$$\sqrt[5]{-\frac{x^5y^{10}z^{15}}{a^{20}b^{15}c^{10}d^5}} = -\frac{xy^2z^3}{a^4b^3c^2d}. \quad \text{Ans.}$$

(266) Using the proportion of Art. **615**,

$$AB : BC = \sin C : \sin A,$$

or

$$70 : 42 = \sin C : \sin 36° \; 10'.$$

Hence, $\sin C = \frac{70}{42} \times \sin 36° \; 10' = \frac{70}{42} \times .59014 = .98357$.

The angle whose sine is .98357 is 79° 36′; hence, angle $C =$ 79° 36′. Ans.

Angle $B = 180° - (A + C) = 180° - (36° \; 10' + 79° \; 36') =$ 64° 14′. Ans.

Using the proportion again,

$AC : BC = \sin B : \sin A$;

or AC : 42 ft. $= \sin 64° \; 14' : \sin 36° \; 10' = .90057 : .59014$.

Hence, $AC = \dfrac{42 \text{ ft.} \times .90057}{.59014} = 64.1$ ft. nearly. Ans.

(267) (*a*) See Art. **591.**

180° − 72° 11′ 36″ = 107° 48′ 24″. Ans.

(*b*) See Art. **590.**

90° − 22° 34′ 17″ = 67° 25′ 43″. Ans.

(268) Angle $B = 180° - (A + C) =$

$180° - (57° \; 34.5' + 44° \; 22.5') = 78° \; 3'$. Ans.

$AC : AB = \sin B : \sin C = \sin 78° \; 3' : \sin 44° \; 22.5'$
$= .97833 : .69936$.

Hence, $AC = \dfrac{344 \text{ ft.} \times .97833}{.69936} = 481.22$ ft. Ans.

$BC : AB = \sin A : \sin C = \sin 57° \; 34.5' : \sin 44° \; 22.5'$
$= .8441 : .69936$.

Hence, $BC = \dfrac{344 \text{ ft.} \times .8441}{.69936} = 415.19$ ft. Ans.

LOGARITHMS.

(QUESTIONS 269-283.)

(269) First raise $\frac{200}{100}$ to the .29078 power. Since $\frac{200}{100} = 2$, $\left(\frac{200}{100}\right)^{.29078} = 2^{.29078}$, and $\log 2^{.29078} = .29078 \times \log 2 = .29078 \times .30103 = .08753$. Number corresponding = 1.2233. Then,

$$1 - \left(\frac{200}{100}\right)^{.29078} = 1 - 1.2233 = -.2233.$$

We now find the product required by adding the logarithms of 351.36, 100, 24, and .2233, paying no attention to the negative sign of .2233 until the product is found. (Art. **647.**)

$$\begin{aligned} \text{Log } 351.36 &= 2.54575 \\ \log \quad 100 &= 2 \\ \log \quad 24 &= 1.38021 \\ \log \quad .2233 &= \underline{\bar{1}.34889} \\ \textit{sum} &= 5.27485 = \end{aligned}$$

$$\log 351.36 \times 100 \times 24 \left[1 - \left(\frac{200}{100}\right)^{.29078}\right]$$

Number corresponding = 188,300.

The number is negative, since multiplying positive and negative signs gives negative; and the sign of .2233 is minus. Hence,

$$x = -188,300. \quad \text{Ans.}$$

(270) (*a*) Log 2,376 = 3.37585. Ans. (See Arts. **625** and **627.**)

(*b*) Log .6413 = $\bar{1}$.80706. Ans.

(*c*) Log .0002507 = $\bar{4}$.39915. Ans.

(271) (*a*) Apply rule, Art. **652.**

$$\begin{array}{rl}
\text{Log } 755.4 = & 2.87818 \\
\text{log } .00324 = & \bar{3}.51055 \\ \hline
\textit{difference} = & 5.36763 = \text{logarithm of quotient.}
\end{array}$$

The mantissa is not found in the table. The next less mantissa is 36754. The difference between this and the next greater mantissa is $773 - 754 = 19$, and the P. P. is $763 - 754 = 9$. Looking in the P. P. section for the column headed 19, we find opposite 9.5, 5, the fifth figure of the number. The fourth figure is 1, and the first three figures 233; hence, the figures of the number are 23315. Since the characteristic is 5, $755.4 \div .00324 = 233{,}150$. Ans.

(*b*) Apply rule, Art. **652.**

$$\begin{array}{rl}
\text{Log } .05555 = & \bar{2}.74468 \\
\text{log } .0008601 = & \bar{4}.93455 \\ \hline
\textit{difference} = & 1.81013 = \text{logarithm of quotient.}
\end{array}$$

The number whose logarithm is 1.81013 equals 64.584.

Hence, $.05555 \div .0008601 = 64.584$. Ans.

(*c*) Apply rule, Art. **652.**

$$\begin{array}{rl}
\text{Log } 4.62 = & .66464 \\
\text{log } .6448 = & \bar{1}.80943 \\ \hline
\textit{difference} = & .85521 = \text{logarithm of quotient.}
\end{array}$$

Number whose logarithm $= .85521 = 7.1648$.

Hence, $4.62 \div .6448 = 7.1648$. Ans.

(272)

$$x^{.74} = \frac{238 \times 1{,}000}{.0042^{.6602}}.$$

$$\begin{array}{rl}
\text{Log } 238 = & 2.37658 \\
\text{log } 1{,}000 = & 3. \\ \hline
\textit{sum} = & 5.37658 = \log (238 \times 1{,}000).
\end{array}$$

$$\begin{array}{rr}
\text{Log } .0042 = & \bar{3}.62325 \\
& .6602 \\ \hline
& 124650 \\
& 373950 \\
& 373950 \\ \hline
& .411469650 \text{ or } .41147.
\end{array}$$

$$\begin{array}{r} .6602 \\ -3 \\ \hline -1.9806 \end{array} = \text{characteristic.}$$

Adding,
$$\begin{array}{r} .41147 \\ -\,1.9806 \\ \hline \bar{2}.43087 \end{array} \quad \text{(See Art. } \mathbf{659.}\text{)}$$

Then, $\log\left(\frac{238 \times 1,000}{.0042^{.6602}}\right) = 5.37658 - \bar{2}.43087 = 6.94571 =$ $.74 \log x$; whence, $\log x = \frac{6.94571}{.74} = 9.38609$. Number whose logarithm $= 9.38609$ is $2,432,700,000 = x$. Ans.

(273)
$$\begin{aligned} \text{Log } .00743 &= \bar{3}.87099 \\ \log .006 &= \bar{3}.77815. \end{aligned}$$

$\sqrt[5]{.00743} = \log .00743 \div 5$ (Art. **662**), and $\sqrt[.6]{.006} = \log .006 \div .6$. Since these numbers are wholly decimal, we apply Art. **663.**

$$\begin{array}{r} 5\,)\,\overline{\bar{3}.87099} \\ \bar{1}.57419 \end{array} = \log \sqrt[5]{.00743}.$$

The characteristic $\bar{3}$ will not contain 5. We then add $\bar{2}$ to it, making $\bar{5}$. 5 is contained in $\bar{5}$, $\bar{1}$ times. Hence, the characteristic is $\bar{1}$. Adding the same number, 2, to the mantissa, we have 2.87099. $2.87099 \div 5 = .57419$. Hence, $\log \sqrt[5]{.00743} = \bar{1}.57419$.

$$\begin{array}{r} .6\,)\,\overline{\bar{3}.77815} \\ \bar{5}. \\ 1.29691 \\ \hline \end{array} \qquad \begin{array}{l} .6 \text{ is contained in } \bar{3}, -5 \text{ times.} \\ .6 \text{ is contained in } .77815,\ 1.29691 \text{ times.} \end{array}$$

$$\textit{sum} = -\bar{4}.29691 = \sqrt[.6]{.006}.$$

$$\begin{aligned} \text{Log } \sqrt[5]{.00743} &= \bar{1}.57419 \\ \log \sqrt[.6]{.006} &= \underline{\bar{4}.29691} \\ \textit{difference} &= 3.27728 = \text{log of quotient.} \end{aligned}$$

Number corresponding $= 1,893.6$.

Hence, $\sqrt[5]{.00743} \div \sqrt[.6]{.006} = 1,893.6$. Ans.

(274) Apply rule. Art. **647.**

$$\begin{aligned}
\text{Log}\ \ 1{,}728 &= 3.23754\\
\log\ .00024 &= \bar{4}.38021\\
\log\ \ .7462 &= \bar{1}.87286\\
\log\ \ 302.1 &= 2.48015\\
\log\ 7.6094 &= \underline{\ \ .88135}\\
\textit{sum} &= 2.85211 =
\end{aligned}$$

log $(1{,}728 \times .00024 \times .7462 \times 302.1 \times 7.6094)$. Number whose logarithm is $2.85211 = 711.40$, the product. Ans.

(275)

$$\begin{aligned}
\text{Log}\ \sqrt{5.954} &= \ .77481 \div 2 = .38741\\
\log \sqrt[3]{61.19} &= 1.78668 \div 3 = \underline{.59556}\\
\textit{sum} &= .98297\\
\log \sqrt[5]{298.54} &= 2.47500 \div 5 = .49500.
\end{aligned}$$

Then,

$\dfrac{\sqrt{5.954} \times \sqrt[3]{61.19}}{\sqrt[5]{298.54}} = \log(\sqrt{5.954} \times \sqrt[3]{61.19}) - \log \sqrt[5]{298.54} =$ $.98297 - .49500 = .48797 =$ logarithm of the required result.

Number corresponding $= 3.0759$. Ans.

(276) $\sqrt[7]{.0532864} = \log .0532864 \div 7$.

Log $.0532864 = \bar{2}.72661$.

Adding $\bar{5}$ to characteristic $\bar{2} = \bar{7}$.

Adding 5 to mantissa $= 5.72662$.

$\bar{7} \div 7 = \bar{1}$.

$5.72661 \div 7 = .81809$, nearly.

Hence, log $\sqrt[7]{.0532864} = \bar{1}.81809$.

Number corresponding to log $\bar{1}.81809 = .65780$. Ans.

(277) (*a*) $32^{4.8}$.

Log $32 = 1.50515$.

$$\begin{array}{r}
1.50515\\
\underline{4.8}\\
1204120\\
\underline{602060}\\
7.224720
\end{array}$$

7.22472 is the logarithm of the required power. (Art. **657.**)

Number whose logarithm $= 7.22472$ is $16{,}777{,}000$.

Hence, $32^{4.8} = 16{,}777{,}000$. Ans.

(*b*) $.76^{3.62}$.

Log $.76 = \bar{1}.88081$.

(See Arts. **658** and **659.**)

$$\begin{array}{r}
\bar{1} + .8\,8\,0\,8\,1 \\
3.6\,2 \\
\hline
1\,7\,6\,1\,6\,2 \\
5\,2\,8\,4\,8\,6 \\
2\,6\,4\,2\,4\,3 \\
\hline
3.1\,8\,8\,5\,3\,2\,2 \\
-\,3.6\,2 \\
\hline
\bar{1}.5\,6\,8\,5\,3 = \log .37028.
\end{array}$$

Hence, $.76^{3.62} = .37028$. Ans.

(*c*) $.84^{.38}$.

Log $.84 = \bar{1}.92428$.

$$\begin{array}{r}
\bar{1} + .9\,2\,4\,2\,8 \\
.3\,8 \\
\hline
7\,3\,9\,4\,2\,4 \\
2\,7\,7\,2\,8\,4 \\
\hline
.3\,5\,1\,2\,2\,6\,4 \\
-\,.3\,8 \\
\hline
\bar{1}.9\,7\,1\,2\,3 = \log .93590.
\end{array}$$

Hence, $.84^{.38} = .93590$. Ans.

(278) $\text{Log}\sqrt[6]{\frac{1}{249}} - \log\sqrt[5]{\frac{23}{71}} =$ logarithm of answer.

$\text{Log}\sqrt[6]{\frac{1}{249}} = \frac{1}{6}(\log 1 - \log 249) = \frac{1}{6}(0 - 2.39620) = -.39937$ $=$ (adding $+1$ and -1) $\bar{1}.60063$.

$\text{Log}\sqrt[5]{\frac{23}{71}} = \frac{1}{5}(\log 23 - \log 71) = \frac{1}{5}(1.36173 - 1.85126) = \frac{1}{5}(-.48953) = -.097906 =$ (adding $+1$ and -1) $\bar{1}.902094$, or $\bar{1}.90209$ when using 5-place logarithms.

Hence, $\bar{1}.60063 - \bar{1}.90209 = \bar{1}.69854 = \log .49950$.

Therefore, $\sqrt[6]{\frac{1}{249}} \div \sqrt[5]{\frac{23}{71}} = .49950$. Ans.

(279) The mantissa is not found in the table. The next less mantissa is .81291; the difference between this and the next greater mantissa is 298 — 291 = 7, and the P. P. is .81293 — .81291 = 2. Looking in the P. P. section for the column headed 7, we find opposite 2.1, 3, the fifth figure of the number; the fourth figure is 0, and the first three figures, 650. Hence, the number whose logarithm is .81293 is .65003. Ans.

2.52460 = logarithm of 334.65. Ans. (See Art. **640.**)

$\overline{1}.27631$ = logarithm of .18893. Ans. We choose 3 for the fifth figure because, in the proportional parts column headed 23, 6.9 is nearer 8 than 9.2.

(280) The most expeditious way of solving this example is the following:

$$p v^{1.41} = p_1 v_1^{1.41}, \text{ or } v_1 = \sqrt[1.41]{\frac{p v^{1.41}}{p_1}} = v \sqrt[1.41]{\frac{p}{p_1}}.$$

Substituting values given, $v_1 = 1.495 \sqrt[1.41]{\dfrac{134.7}{16.421}}$.

$$\text{Log } v_1 = \log\ 1.495 + \frac{\log\ 134.7 - \log\ 16.421}{1.41} = .17464 +$$

$$\frac{2.12937 - 1.21540}{1.41} = .17464 + .64821 = .82285 = \log\ 6.6504;$$

whence, $v_1 = 6.6504$. Ans.

(281) $\text{Log} \sqrt[5]{\dfrac{7.1895 \times 4{,}764.2^2 \times 0.00326^5}{.000489 \times 457^3 \times .576^2}} = \frac{1}{5}[\log 7.1895$ $+ 2 \log 4{,}764.2 + 5 \log .00326 - (\log .000489 + 3 \log 457 +$ $2 \log .576)] = \dfrac{\overline{5}.77878 - 4.18991}{5} = \overline{2}.31777 = \log .020786.$ Ans.

$$\begin{aligned} \text{Log } 7.1895 &= .85670 \\ 2 \log 4{,}764.2 = 2 \times 3.67799 &= 7.35598 \\ 5 \log .00326 = 5 \times \overline{3}.51322 &= \underline{\overline{13}.56610} \\ sum &= \overline{5}.77878. \end{aligned}$$

$$\begin{aligned} \text{Log } .000489 &= \overline{4}.68931 \\ 3 \log 457 = 3 \times 2.65992 &= 7.97976 \\ 2 \log .576 = 2 \times \overline{1}.76042 &= \underline{\overline{1}.52084} \\ sum &= 4.18991. \end{aligned}$$

(282) Substituting the values given,

$$p = \frac{\overset{8,000}{\cancel{960,000}} \times \left(\frac{3}{16}\right)^{2.18}}{\cancel{120} \times 2.25} = \frac{8,000\left(\frac{3}{16}\right)^{2.18}}{2.25}.$$

Log p = log 8,000 + 2.18 log $\frac{3}{16}$ − log 2.25 = 3.90309 + 2.18 (log 3 − log 16) − .35218 = 3.55091 + 2.18 × (.47712 − 1.20412) = 1.96605 = log 92.480. Ans.

(283) Solving for t, $t = \sqrt[2.18]{\frac{p\,l\,d}{960,000}}$.

Substituting values given,

$$t = \sqrt[2.18]{\frac{\overset{44}{\cancel{160} \times \cancel{132} \times \cancel{2}}}{\underset{\underset{\underset{1,000}{\cancel{3,000}}}{\cancel{6,000}}}{\cancel{960,000}}}} = \sqrt[2.18]{.044}.$$

$$\text{Log } t = \frac{\log .044}{2.18} = \frac{\bar{2}.64345}{2.18} = \frac{-2.18 + .82345}{2.18} =$$

$$\bar{1}.37773 = \log .23863. \quad \text{Ans.}$$

ELEMENTARY MECHANICS.

(QUESTIONS 355-453.)

(355) Use formulas **18** and **8.**

Time it would take the ball to fall to the ground $= t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5.5}{32.16}} = .58484$ sec.

The space passed through by a body having a velocity of 500 ft. per sec. in .58484 of a second $= S = Vt = 500 \times .58484 = 292.42$ ft. Ans.

(356) Use formula **7.**

$$\frac{\frac{80}{12} \times 3.1416 \times 160}{60} = 55.85 \text{ ft. per sec.} \quad \text{Ans.}$$

(357) $160 \div 60 \times 7 = \frac{8}{21}$ revolution in $\frac{1}{7}$ sec. $360° \times \frac{8}{21} = 137\frac{1}{7}° = 137° \, 8' \, 34\frac{2}{7}''$. Ans.

(358) (*a*) See Fig. 20. $36'' = 3'$. $4 \div 3 = \frac{4}{3} =$ number of revolutions of pulley to one revolution of fly-wheel. $54 \times \frac{4}{3} = 72$ revolutions of pulley and drum per min. $100 \div \left(\frac{18}{12} \times 3.1416\right) = 21.22$ revolutions of drum to raise elevator 100 ft. $\frac{21.22}{72} \times 60 = 17.68$ sec. to travel 100 ft. Ans.

(*b*) $21.22 : x :: 30 : 60$, or $x = \frac{21.22 \times 60}{30} = 42.44$ rev.

per min. of drum. The diameter of the pulley divided by the diameter of the fly-wheel $= \frac{3}{4}$, which multiplied by $42.44 = 31.83$ revolutions per min. of fly-wheel. Ans.

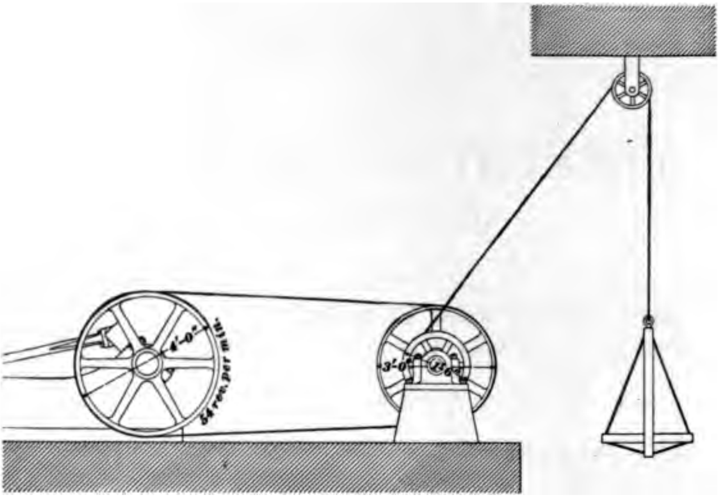

FIG. 20.

(359) See Arts. **857** and **859.**

(360) See Art. **861.**

(361) See Arts. **843** and **871.**

(362) See Art. **871.**

(363) See Arts. **842, 886, 887,** etc.

The relative weight of a body is found by comparing it with a given standard by means of the balance. The absolute weight is found by noting the pull which the body will exert on a spring balance.

The absolute weight increases and decreases according to the laws of weight given in Art. **890**; the relative weight is always the same.

(364) See Art. **861.**

(365) See Art. **857.**

(366) See Art. **857.**

(367) If the mountain is at the same height above, and the valley at the same depth below sea-level respectively, it will weigh more at the bottom of the valley.

(368) $\frac{31,680}{5,280} = 6$ miles. Using formula **12,** $d^2 : R^2 ::$ $W : w$, we have $w = \frac{R^2 W}{d^2} = \frac{3,960^2 \times 20,000}{3,966^2} = 19,939.53 +$ lb. $= 19,939$ lb. $8\frac{1}{2}$ oz. Ans.

(369) Using formula **11,** $R : d :: W : w$, we have $w = \frac{d W}{R} = \frac{3,958 \times 20,000}{3,960} = 19,989.89$ lb. $= 19,989$ lb. $14\frac{1}{4}$ oz. Ans.

(370) See Art. **870.**

(371) See Art. **894.**

(372) The velocity which a body may have at the instant the time begins to be reckoned.

(373) Because the man after jumping tends to continue in motion with the same velocity as the train, and the sudden stoppage by the earth causes a shock, the severity of which varies with the velocity of the train.

(374) See Arts. **870** and **871.**

(375) See Art. **872.**

(376) That force which will produce the same final effect upon a body as all the other forces acting together is called the resultant.

(377) (*a*) If a 5-in. line = 20 lb., a 1-in. line = 4 lb. $1 \div 4 = \frac{1}{4}$ in. = 1 lb. Ans. (*b*) $6\frac{1}{4} \div 4 = 1.5625$ in. = $6\frac{1}{4}$ lb. Ans.

(378) Those forces by which a given force may be replaced, and which will produce the same effect upon a body.

(379) Southeast, in the direction of the diagonal of a square. See Fig. 21.

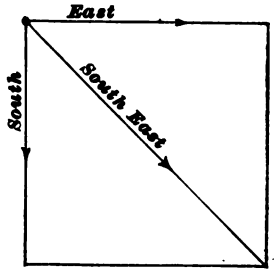

FIG. 21.

(380) $4'\ 6'' = 54''$. $54 \times 2 \times \frac{3}{4} \times .261 = 21.141$ lb. = weight of lever. Center of gravity of lever is in the middle, at a, Fig. 22, 27″ from each end. Consider that the lever has no weight. The center of gravity of the two weights is at b, at a distance from c equal to $\frac{47 \times 54}{47 + 71} = 21.508'' = bc$. Formula **20**, Art. **911**.

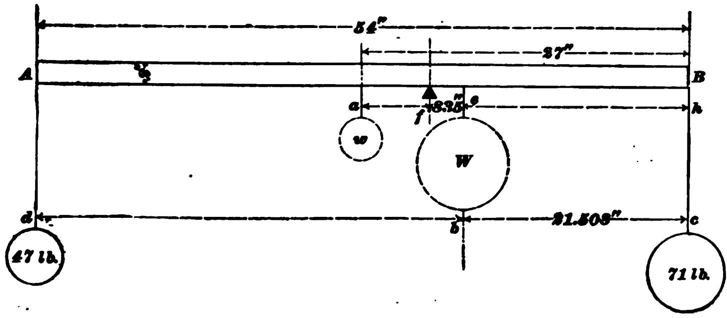

FIG. 22.

Consider both weights as concentrated at b, that is, imagine both weights removed and replaced by the dotted weight W, equal to $71 + 47 = 118$ lb. Consider the weight of the bar as concentrated at a, that is, as if replaced by a weight $w = 21.141$ lb. Then, the distance of the balancing point f, from e, or fe, $= \frac{21.141 \times 5.492}{21.141 + 118} = .835''$, since $ae = 27 - 21.508 = 5.492''$. Finally, $fe + eh = fh = .835 + 21.508 = 22.343''$ = the short arm. Ans. $54 - 22.343 = 31.657''$ = long arm. Ans.

(381) See Fig. 23.

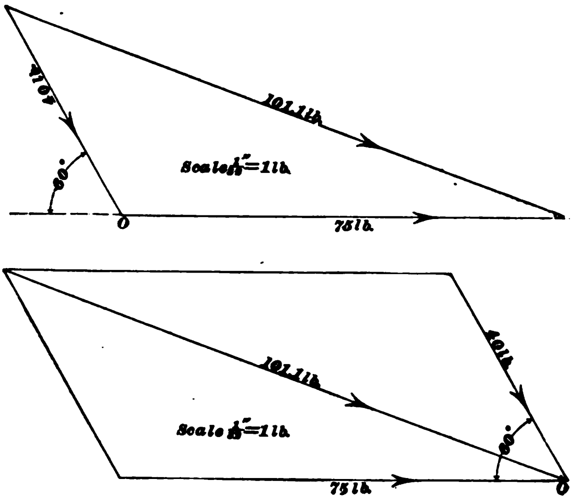

FIG. 23.

(382) See Fig. 24.

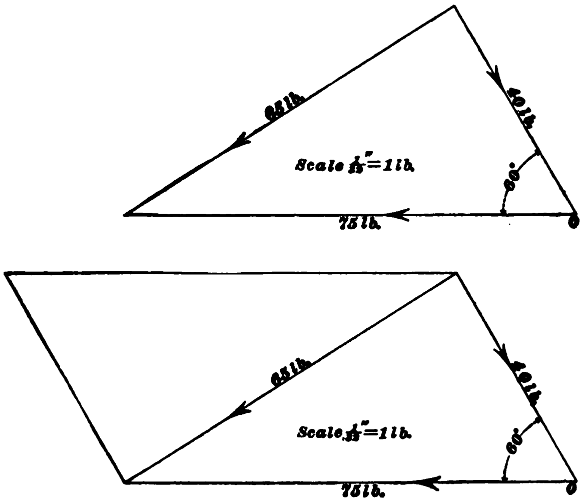

FIG. 24.

(383) 46 − 27 = 19 lb., acting in the direction of the force of 46 lb. Ans.

(384) (*a*) 18 × 60 × 60 = 64,800 miles per hour. Ans.
(*b*) 64,800 × 24 = 1,555,200 miles. Ans.

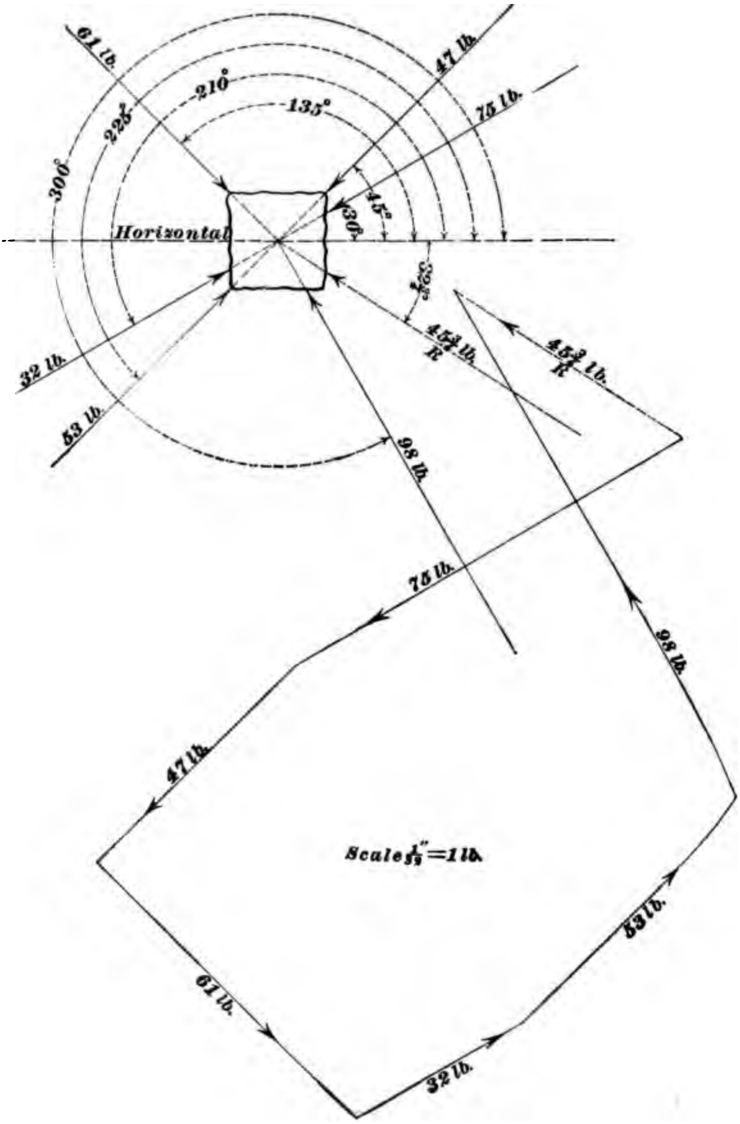

FIG. 25.

(385) (*a*) 15 miles per hour $= \frac{15 \times 5,280}{60 \times 60} = 22$ ft. per sec. As the other body is moving 11 ft. per sec., the distance between the two bodies in one second will be $22 + 11 = 33$ ft., and in 8 minutes the distance between them will be $33 \times 60 \times 8 = 15,840$ ft., which, divided by the number of feet in one mile, gives $\frac{15,840}{5,280} = 3$ miles. Ans.

(*b*) As the distance between the two bodies increases 33 ft. per sec., then, 825 divided by 33 must be the time required for the bodies to be 825 ft. apart, or $\frac{825}{33} = 25$ sec. Ans.

(386) See Fig. 25.

(387) (*a*) Although not so stated, the velocity is evidently considered with reference to a point on the shore. $10 - 4 = 6$ miles an hour. Ans.

(*b*) $10 + 4 = 14$ miles an hour. Ans.

(*c*) $10 - 4 + 3 = 9$, and $10 + 4 + 3 = 17$ miles an hour. Ans.

(388) See Fig. 26.

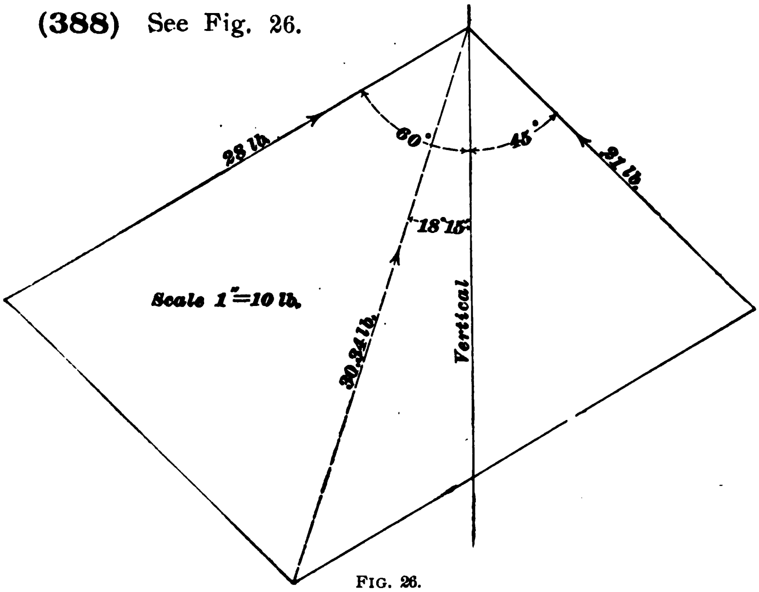

FIG. 26.

(398) 1 hour = 60 min., 1 day = 24 hours; hence, 1 day = 60 × 24 = 1,440 min. Using formula **7**, $V = \frac{S}{t}$; whence, $V = \frac{8,000 \times 3.1416}{1,440} = 17.453+$ miles per min. Ans.

(399) (*a*) Use formula **25**.

Kinetic energy $= \frac{Wv^2}{2g} = \frac{400 \times 1,875 \times 1,875}{2 \times 32.16} = 21,863,339.55$ ft.-lb. Ans.

(*b*) $\frac{21,863,339.55}{2,000} = 10,931.67$ ft.-tons. Ans.

(*c*) See Art. **961**.

Striking force $\times \frac{6}{12} = 21,863,339.55$ ft.-lb.,

or striking force $= \frac{21,863,339.55}{\frac{6}{12}} = 43,726,679$ lb. Ans.

(400) Using formula **18**, $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 200}{32.16}} =$ 3.52673 sec., when $g = 32.16$.

$t = \sqrt{\frac{2 \times 200}{20}} = 4.47214$ sec., when $g = 20$.

$4.47214 - 3.52673 = 0.94541$ sec. Ans.

(401) See Art. **910**.

(402) See Art. **963**.

(403) (*a*) See Art. **962**.

$D = \frac{m}{V} = \frac{W}{gv}$. $v = \frac{800}{1,728}$. Hence, $D = \frac{W}{gv} = \frac{500}{32.16 \times \frac{800}{1728}} =$ 33.582. Ans. (*b*) In Art. **962**, the density of water was found to be 1.941. (*c*) In Art. **963**, it is stated that the specific gravity of a body is the ratio of its density to the density of water. Hence, $\frac{33.582}{1.941} = 17.3 =$ specific gravity. If the weight of water be taken as 62.5 lb. per cu. ft., the specific gravity will be found to be 17.28. Ans.

(404) Assuming that it started from a state of rest, formula **13** gives $v = g\,t = 32.16 \times 5 = 160.8$ ft. per sec.

(405) Use formulas **17** and **13.** $h = \frac{1}{2} g t^2 = \frac{32.16}{2} \times 3^2 = 144.72$ ft., distance fallen at the end of third second.

$v = g t = 32.16 \times 3 = 96.48$ ft. per sec., velocity at end of third second.

$96.48 \times 6 = 578.88$ ft., distance fallen during the remaining 6 seconds.

$144.72 + 578.88 = 723.6$ ft. = total distance. Ans.

(406) See Art. **961.**

Striking force $\times \frac{\frac{1}{2}}{12} = 8 \times 8 = 64$. Therefore, striking force $= \frac{64}{\frac{\frac{1}{2}}{12}} = 1,536$ tons. Ans.

(407) See Arts. **901** and **902.**

(408) Use formula **19.**

Centrifugal force = tension of string $= .00034\ W R N^2 = .00034 \times (.5236 \times 4^3 \times .261) \times \frac{15}{12} \times 60^2 = 13.38 +$ lb. Ans.

(409) $(80^2 - 70^2) \times .7854 \times 26 \times .261 \times \frac{1}{2} = 3,997.2933$ lb., weight of one-half the fly-wheel rim. Inside radius $= \frac{80 - 10}{2} = 35$ in., or $\frac{35}{12} = 2\frac{11}{12}$ ft.

According to formula **19,** $F = .00034\ W R N^2 =$

$$\frac{.00034 \times 3,997.2933 \times 35 \times 175^2}{12} = 121,394+ \text{ lb. Ans.}$$

(410) (*a*) Use formulas **11** and **12.** $R : d :: W : w$, or $W = \frac{w R}{d} = \frac{1 \times 4,000}{100} = 40$ lb. Ans.

(*b*) $d^2 : R^2 :: W : w$, or $w = \frac{4,000^2 \times 40}{4,100^2} = 38.072$ lb. Ans.

(411) See Art. **955.**

$$\frac{10,746 \times 354}{10 \times 33,000} = 11.5275 \text{ H. P. Ans.}$$

(412) Use formula **12.**

$d^2 : R^2 :: W : w$, or $d = \sqrt{\dfrac{4{,}000^2 \times 2}{\frac{3}{16}}} = 13{,}064$ mi., nearly.

$13{,}064 - 4{,}000 = 9{,}064$ miles. Ans.

(413) Use formula **18.** $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 50}{32.16}} =$ 1.7634 sec., nearly.

$1.7634 \times 140 = 246.876$ ft. Ans.

(414) $\dfrac{10}{30} = \dfrac{1}{3}$ sec. Use formula **17.**

$h = \dfrac{1}{2} g t^2 = \dfrac{1}{2} \times 32.16 \times \left(\dfrac{1}{3}\right)^2 = 1.78\frac{2}{3}$ ft. $= 1$ ft. 9.44 in. Ans.

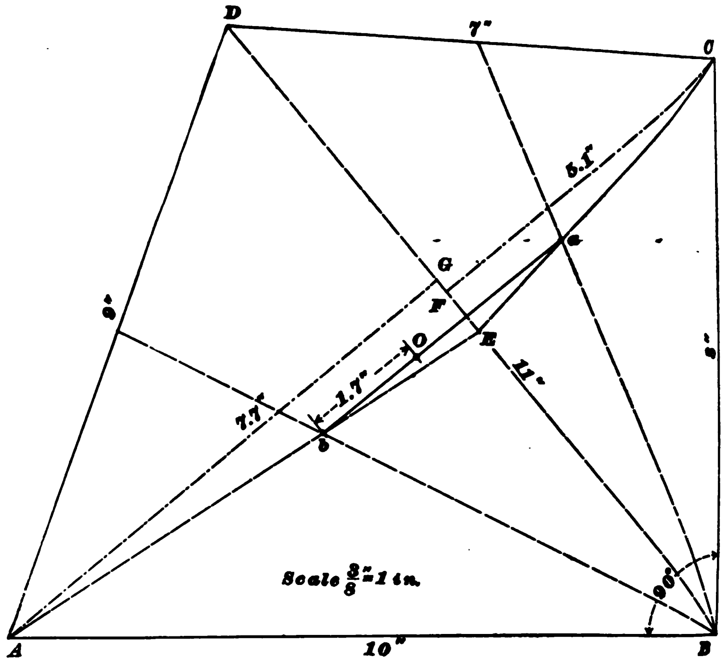

FIG. 29.

(415) See Arts. **906, 907.**

(416) See Arts. **908, 909.**

(417) No. It can only be counteracted by another equal couple which tends to revolve the body in an opposite direction.

(418) See Art. **914.**

(419) Draw the quadrilateral as shown in Fig. 29. Divide it into two triangles by the diagonal *B D*. The center of gravity of the triangle *B C D* is found to be at *a*, and the center of gravity of the triangle *A B D* is found to be at *b* (Art. **914**). Join *a* and *b* by the line *a b*, which, on being measured, is found to have a length of 4.27 inches. From *C* and *A* drop the perpendiculars *C F* and *A G* on the diagonal *B D*. Then, area of the triangle $A\,B\,D = \frac{1}{2}(A\,G \times B\,D)$, and area of the triangle $B\,C\,D = \frac{1}{2}(C\,F \times B\,D)$. Measuring these distances, $B\,D = 11''$, $C\,F = 5.1''$, and $A\,G = 7.7''$.

$$\text{Area } A\,B\,D = \frac{1}{2} \times 7.7 \times 11 = 42.35 \text{ sq. in.}$$

$$\text{Area } B\,C\,D = \frac{1}{2} \times 5.1 \times 11 = 28.05 \text{ sq. in.}$$

According to formula **20,** the distance of *O*, the center of gravity, from *b* is $\frac{28.05 \times 4.27}{28.05 + 42.35} = 1.7$. Therefore, the center of gravity is on the line *a b* at a distance of 1.7″ from *b*.

(420) See Fig. 30. The center of gravity lies at the geometrical center of the pentagon, which may be found as follows: From any vertex draw a line to the middle point of the opposite side. Repeat the operation for any other vertex, and the intersection of the two lines will be the desired center of gravity.

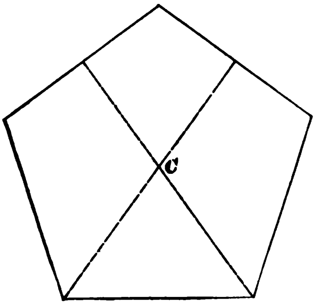

FIG. 30.

(421) See Fig. 31. Since any number of quadrilaterals can be drawn with the sides given, any number of answers can be obtained.

Draw a quadrilateral, the lengths of whose sides are equal to the distances between the weights, and locate a weight on each corner. Apply formula **20** to find the distance $C_1 W_1$; thus, $C_1 W_1 = \frac{9 \times 18}{9 + 21} = 5.4''$. Measure the distance $C_1 W_2$; suppose it equals say 36″. Apply the formula again. $C_1 C_2 = \frac{15 \times 36}{15 + (9 + 21)} = 12''$. Measure $C_2 W_3$; it equals say 31.7″.

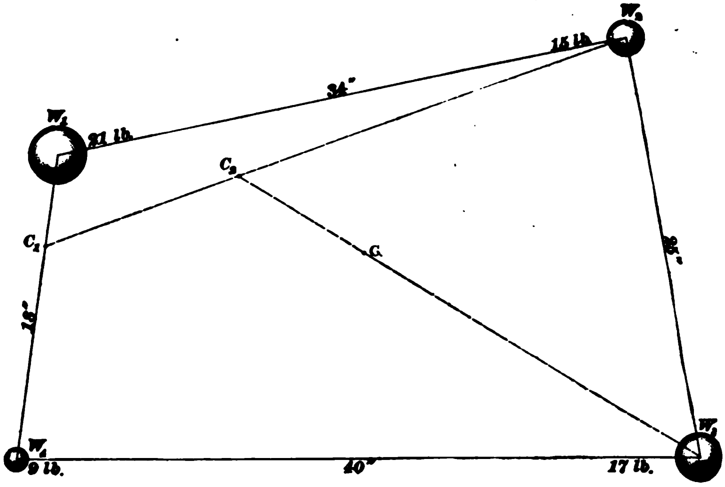

FIG. 31.

Apply the formula again. $C_2 C = \frac{17 \times 31.7}{17 + 15 + 9 + 21} = 8.7''$. C is center of gravity of the combination.

(422) Let $A\ B\ C\ D\ E$, Fig. 32, be the outline, the right-angled triangle cut-off being $E\ S\ D$. Divide the figure into two parts by the line $m\ n$, which is so drawn that it cuts off an isosceles right-angled triangle $m\ B\ n$, equal in area to $E\ S\ D$, from the opposite corner of the square.

The center of gravity of $AmnCDE$ is then at C_1, its geometrical center. $Bm = 4$ in.; angle $Bmr = 45°$; therefore, $Br = Bm \times \sin Bmr = 4 \times .707 = 2.828$ in. C_2, the center of gravity of Bmn, lies on Br, and $BC_2 = \frac{2}{3}Br = \frac{2}{3} \times 2.828 = 1.885$ in. $BC_1 = AB \times \sin BAC_1 = 14 \times \sin 45° = 14 \times .707 = 9.898$ in. $C_1C_2 = BC_1 - BC_2 = 9.898 - 1.885 = 8.013$ in.

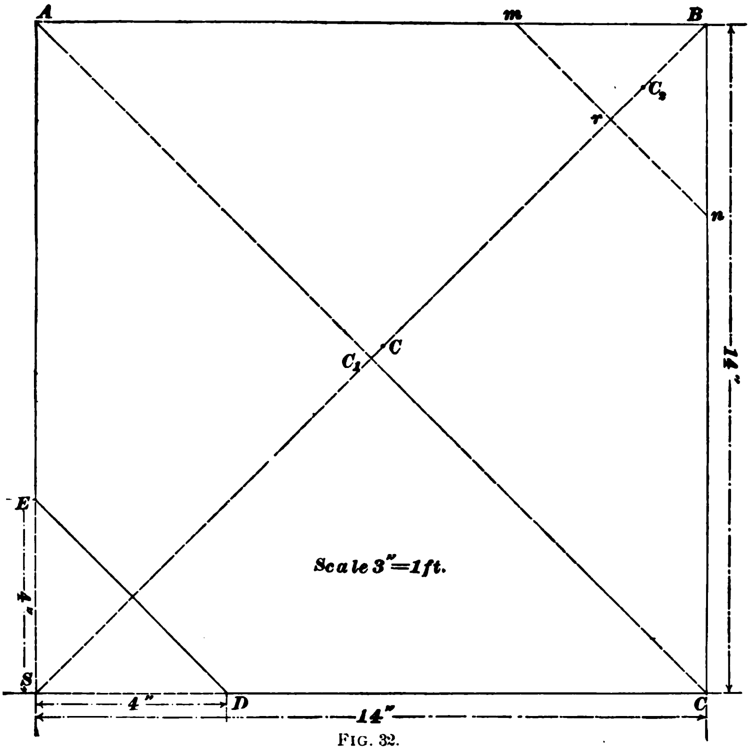

FIG. 32.

Area $ABCDE = 14^2 - \frac{4 \times 4}{2} = 188$ sq. in. Area $mBn = \frac{4 \times 4}{2} = 8$ sq. in. Area $AmnCDE = 188 - 8 = 180$ sq. in.

The center of gravity of the combined area lies at C, at [illegible] 911),

[illegible]

[illegible] ANS.

423. [illegible] will have moved [illegible] and the [illegible]. The velocity ratio is the [illegible]

[illegible] ANS.

[illegible] ANS.

[illegible] ANS.

424. [illegible] 920 and 922.

425. [illegible] Fig. 33. From [illegible] the centre of gravity of the

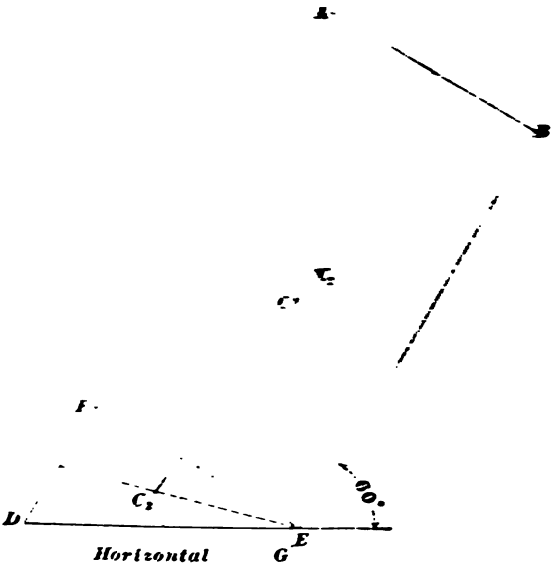

FIG. 33.

rectangle, which is at C_1, and that of the triangle, which is at C_2. Connect these centers of gravity by the straight line $C_1 C_2$ and find the common center of gravity of the body by the rule to be at C. Having found this center, draw the line of direction $C G$. If this line falls within the base, the body will stand, and if it falls without, it will fall.

(426) (*a*) 5 ft. 6 in. = 66″. 66 ÷ 6 = 11 = velocity ratio. Ans.

(*b*) 5 × 11 = 55 lb. Ans.

(427) 55 × .65 = 35.75 lb. Ans.

(428) Apply formula **20.** 5 ft. = 60″. $\frac{35 \times 60}{180 + 35} =$ 9.7674 in., nearly, = distance from the large weight. Ans.

(429) (*a*) 1,000 ÷ 50 = 20, velocity ratio. Ans. See Art. **945.** (*b*) 10 fixed and 10 movable. Ans. (*c*) 50 ÷ 95 = 52.63%. Ans.

(430) $P \times$ circumference $= W \times \frac{1}{8}$, or $60 \times 40 \times 3.1416 = W \times \frac{1}{8}$, or $W = 60 \times 40 \times 3.1416 \times 8 = 60{,}318.72$ lb. Since the efficiency of combination is 40%, the tension on the stud would be .40 × 60,318.72 = 24,127.488 lb. Ans.

(431) (*a*) $\sqrt{20^2 + 5^2} = 20.616$ ft. = length of inclined plane.

$P \times$ length of plane $= W \times$ height, or $P \times 20.616 = 1{,}580 \times 5$.

$P = \frac{1{,}580 \times 5}{20.616} = 383.2$ lb. Ans. (*b*) In the second case, $P \times$ length of base $= W \times$ height, or $P \times 20 = 1{,}580 \times 5$; hence, $P = \frac{1{,}580 \times 5}{20} = 395$ lb. Ans.

(432) $W \times 2 = 42 \times 6$, or $W = \frac{42 \times 6}{2} = 126$ lb.

126 + 42 = 168 lb. $168 \times 1 = W' \times 12$, or $W' = \frac{168}{12} = 14$ lb. Ans.

(433) See Fig. 34. $P \times 14 \times 21 \times 19 = 2\frac{1}{2} \times 3\frac{1}{4} \times 2\frac{7}{8} \times 725$, or

$$P = \frac{2\frac{1}{2} \times 3\frac{1}{4} \times 2\frac{7}{8} \times 725}{14 \times 21 \times 19} = 3.032 \text{ lb.} \quad \text{Ans.}$$

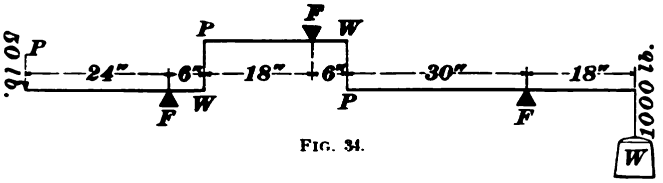

FIG. 34.

(434) See Fig. 35. (*a*) $35 \times 15 \times 12 \times 20 = 5 \times 3\frac{1}{2} \times 3 \times W$, or

$$W = \frac{35 \times 15 \times 12 \times 20}{5 \times 3\frac{1}{2} \times 3} = 2{,}400 \text{ lb.} \quad \text{Ans.}$$

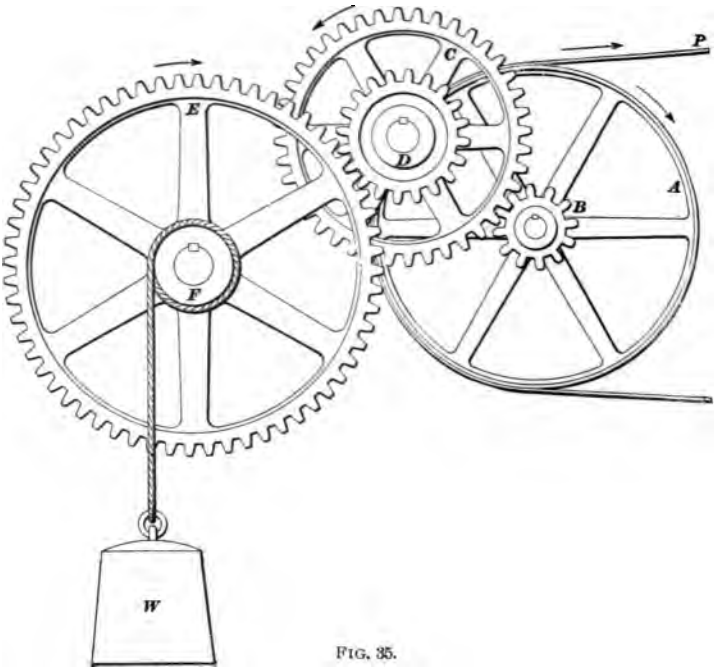

FIG. 35.

(*b*) $2{,}400 \div 35 = 68\frac{4}{7}$ = velocity ratio. Ans.

(*c*) $1{,}932 \div 2{,}400 = .805 = 80.5\%$. Ans.

(**435**) In Fig. 36, let the 12-lb. weight be placed at *A*, the 18-lb. weight at *B*, and the 15-lb. weight at *D*.

Use formula **20.**

$\frac{12 \times 15}{18 + 12} = 6''$ = distance $C_1 B$ = distance of center of gravity of the 12 and 18-lb. weights from *B*. Drawing $C_1 D$, $C_1 C = \frac{15 \times C_1 D}{(12 + 18) + 15} = \frac{1}{3} C_1 D$. Measuring the distances of *C* from *B D*, *D A*, and *A B*, it is found that $C a = 3.45''$, $C b = 5.25''$, and $C d = 4.4''$. Ans.

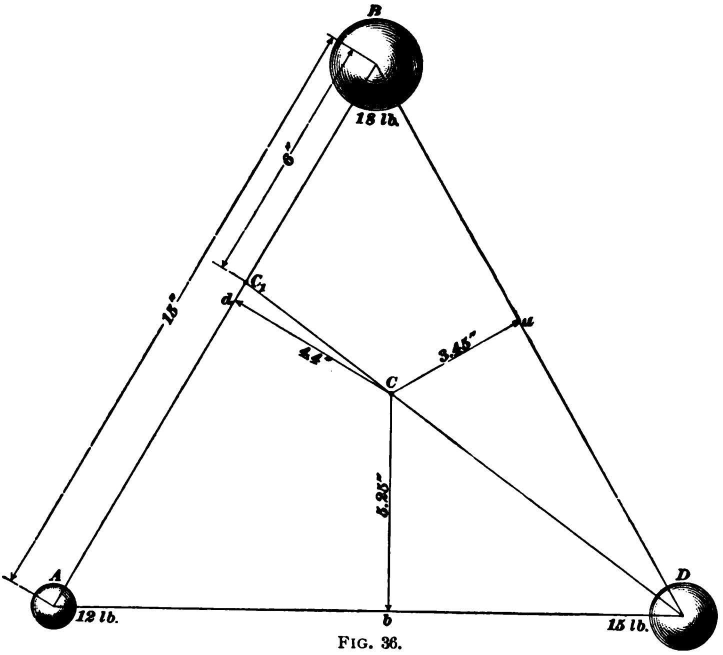

FIG. 36.

(**436**) (*a*) Potential energy equals the work which the body would do in falling to the ground = $500 \times 75 = 37{,}500$ ft.-lb. Ans.

(*b*) Using formula **18**, $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 75}{32.16}} = 2.1597$ sec. = .035995 min., the time of falling.

$$\frac{37,500}{33,000 \times .035995} = 31.57 \text{ H. P. Ans.}$$

(437) $127 \div 62.5 = 2.032$ = specific gravity. Ans.

(438) $\frac{62.5}{1,728} \times 9.823 = .35529$ lb. Ans.

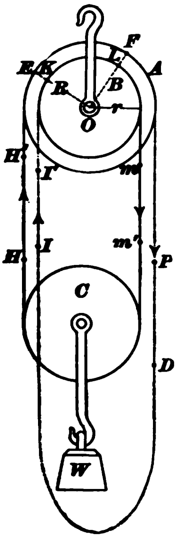

FIG. 37.

(439) Use formula **21.** $W = \left(\frac{2PR}{R - r}\right)$, or $\frac{2 \times 60 \times 6.5}{6.5 - 5.75} \times .48 = 499.2$ lb. Ans. See Fig. 37.

(440) See Art. **961.**

$$F \times \left(\frac{3}{8} \div 12\right) = \frac{Wv^2}{2g} = \frac{1.5 \times 25^2}{2 \times 32.16}, \text{ or}$$

$$F = \frac{\frac{1.5 \times 25^2}{2 \times 32.16}}{\frac{3}{8} \div 12} = 466.42 \text{ lb. Ans.}$$

(441) (*a*) $2,000 \div 4 = 500$ = wt. of cu. ft. $500 \div 62.5 = 8$ = specific gravity. Ans.

(*b*) $\frac{500}{1,728} = .28935$ lb. Ans.

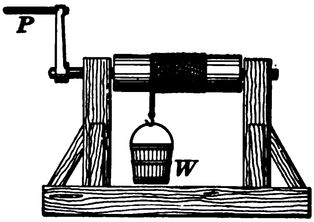

FIG. 38.

(442) See Fig. 38. $14.5 \times 2 = 29$. $30 \times 29 = W \times 5$, or $W = \frac{30 \times 29}{5} = 174$ lb. Ans.

(443) $75 \times .21 = 15.75$ lb. Ans.

(444) (*a*) $900 \times 150 = 135,000$ ft.-lb. Ans.

$$\frac{135,000}{15} = 9,000 \text{ ft.-lb. per min. Ans.}$$

(*b*) $\frac{9,000}{33,000} = \frac{3}{11}$ H.P. Ans.

(445) 900 × .18 × 2 = 324 lb. = force required to overcome the friction. 900 + 324 = 1,224 lb. = total force.

$$\frac{1,224 \times 150}{15 \times 33,000} = .37091 \text{ H. P. Ans.}$$

(446) 18 ÷ 88 = .2045. Ans.

(447) See Art. **962.** $D = \frac{W}{g\,V} = \frac{1,200}{32.16 \times 3} = 12.438.$ Ans.

(448) See Fig. 39. 125 − 47.5 = 77.5 lb. = downward pressure.

77.5 ÷ 4 = 19.375 lb. = pressure on each support. Ans.

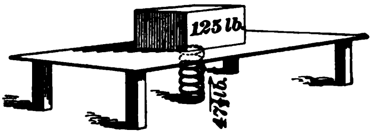

FIG. 39.

(449) See Fig. 40.

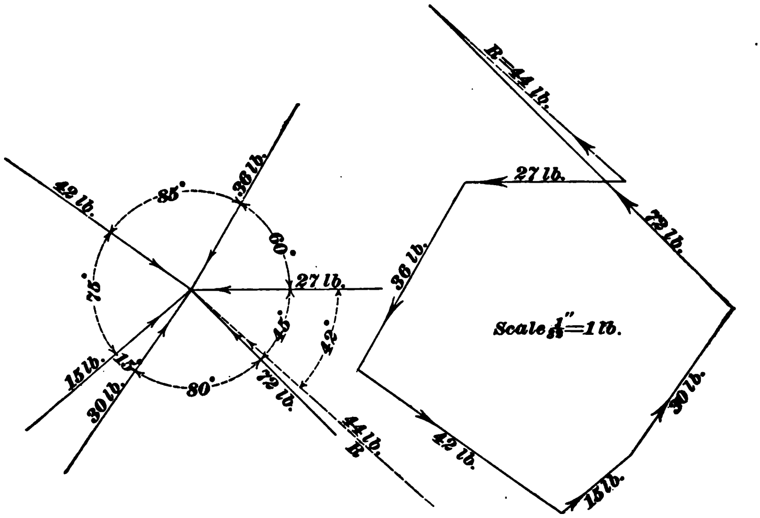

FIG. 40.

(450) See Fig. 41. 4.5 ÷ 2 = 2.25.

$$\frac{12}{2.25} \times 6 \times 30 = 960 \text{ lb. Ans.}$$

(451) (*a*) 960 ÷ 30 = 32. Ans.

(*b*) 790 ÷ 960 = .8229 = 82.29%. Ans.

(452) (*a*) See Fig. 42. $475 + (475 \times .24) = 589$ lb. Ans

(*b*) $475 \div 589 = .8064 = 80.64\%$. Ans.

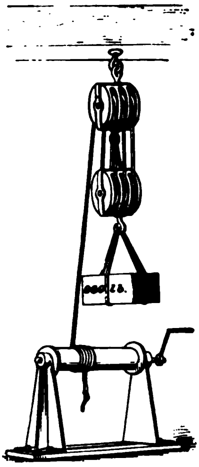

FIG. 41.

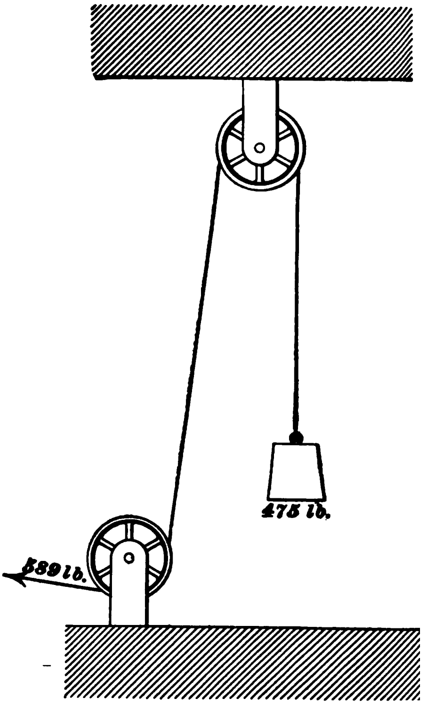

FIG. 42.

(453) (*a*) By formula **23,** $U = FS = 6 \times 25 = 15$ foot-pounds. Ans.

$$(b)\ 2\frac{1}{2} \text{ sec.} = \frac{2\frac{1}{2}}{60} = \frac{1}{24} \text{ min.}$$

Using formula **24,** Power $= \frac{FS}{T} = \frac{150}{\frac{1}{24}} = 3{,}600$ ft.-lb. pe min. Ans.

PNEUMATICS.

(QUESTIONS 504–553.)

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(504) The force with which a confined gas presses against the walls of the vessel which contains it.

(505) (*a*) $4 \times 12 \times .49 = 23.52$ lb. per sq. in. Ans.
(*b*) $23.52 \div 14.7 = 1.6$ atmospheres. Ans.

(506) (*a*) A column of water 1 foot high exerts a pressure of .434 lb. per sq. in. Hence, $.434 \times 19 = 8.246$ lb. per sq. in., the required tension. A column of mercury 1 in. high exerts a pressure of .49 lb. per sq. in. Hence, $8.246 \div .49 = 16.828$ in. = height of the mercury column. Ans.

(*b*) Pressure above the mercury $= 14.7 - 8.246 = 6.454$ lb. per sq. in. Ans.

(507) Using formula **53,** $p_1 = \frac{p\,v}{v_1} = \frac{(14.7 \times 3) \times 1}{2.5} =$ 17.64 lb. Ans.

(508) (*c*) Using formula **61,**

$$V = \frac{.37052\,W\,T}{p} = \frac{.37052 \times 7.14 \times 535}{22.05} = 64.188 \text{ cu. ft. Ans.}$$

($T = 460° + 75° = 535°$, and $p = 14.7 \times 1.5 = 22.05$ lb. per sq. in.)

(*a*) $7.14 \div .08 = 89.25$ cu. ft., the original volume. Ans.

(*b*) If 1 cu. ft. weighs .08 lb., 1 lb. contains $1 \div .08 = 12.5$ cu. ft. Hence, using formula **60,** $p\,V = .37052\,T$, or $T = \frac{p\,V}{.37052} = \frac{22.05 \times 12.5}{.37052} = 743.887°$. $743.887 - 460 = 283.887°$. Ans.

(509) Substituting in formula **59**, $p = 40$, $t = 120$, and $t_1 = 55$,

$$p_1 = 40\left(\frac{460 + 55}{460 + 120}\right) = \frac{40 \times 515}{580} = 35.517 \text{ lb.} \quad \text{Ans.}$$

(510) Using formula **61**, $p\,V = .37052\, W\,T$, or

$$W = \frac{p\,V}{.37052\,T}. \quad T = 460° + 60° = 520°.$$

Therefore, $W = \dfrac{14.7 \times 1}{.37052 \times 520} = .076296$ lb. Ans.

(511) $175{,}000 \div 144 =$ pounds per sq. in.

$(175{,}000 \div 144) \div 14.7 = 82.672 =$ atmospheres. Ans.

(512) Extending formula **63** to include 3 gases, we have $P\,V = p_1 v_1 + p_2 v_2 + p_3 v_3$, or $40 \times P = 1 \times 12 + 2 \times 10 + 3 \times 8$. Hence, $P = \dfrac{56}{40} = 1.4$ atmos. $= 1.4 \times 14.7 = 20.58$ lb. per sq. in. Ans.

(513) In the last example, $P\,V = 56$. In the present case, $P = \dfrac{23}{14.7}$ atmos. Therefore, $V = \dfrac{56}{P} = \dfrac{56}{\frac{23}{14.7}} = 35.79$ cu. ft. Ans.

(514) For $t = 280°$, $T = 740°$; for $t = 77°$, $T = 537°$.

$p\,V = .37052\, W\,T$, or $W = \dfrac{p\,V}{.37052\,T}$. (Formula **61.**)

$$\text{Weight of hot air} = \frac{14.7 \times 10{,}000}{.37052 \times 740} = 536.13 \text{ lb.}$$

$$\text{Weight of air displaced} = \frac{14.7 \times 10{,}000}{.37052 \times 537} = 738.81 \text{ lb.}$$

$738.81 - 536.13 = 202.68$. $202.68 - 100 = 102.68$ lb. Ans.

(515) According to formula **64**,

$$P\,V = \left(\frac{p_1 v_1}{T_1} + \frac{p_2 v_2}{T_2}\right) T, \text{ or}$$

$$20 \times 31 = \left(\frac{14.7 \times 13}{533} + \frac{30 \times 18}{513}\right) T = 1.411168\, T.$$

Therefore, $T = \frac{20 \times 31}{1.411168} = 439.35°$. Since this is less than 460°, the temperature is $460 - 439.35 = 20.65°$ below zero, or $-20.65°$. Ans.

(516) A hollow space from which all air or other gas (or gaseous vapor) has been removed. An example would be the space above the mercury in a barometer.

(517) One inch of mercury corresponds to a pressure of .49 lb. per sq. in.

$\frac{1}{40}$ inch of mercury corresponds to a pressure of $\frac{.49}{40}$ lb. per sq. in. $\frac{.49}{40} \times 144 = 1.764$ lb. per sq. ft. Ans.

(518) (*a*) $325 \times .14 = 45.5$ lb. = force necessary to overcome the friction. $6 \times 12 = 72'' =$ length of cylinder. $72 - 40 = 32 =$ distance which the piston must move. Since the area of the cylinder remains the same, any variation in the volume will be proportional to the variation in the length between the head and piston. By formula **53,** $p\,v = p_1\,v_1$. Therefore, $p = \frac{p_1\,v_1}{v} = \frac{14.7 \times 40}{72} = 8.1\frac{2}{3}$ lb. per sq. in. = pressure when piston is at the end of the cylinder. Since there is the atmospheric pressure of 14.7 lb. on one side of the piston and only $8.1\frac{2}{3}$ lb. on the other side, the force required to pull it out of the cylinder is $14.7 - 8.1\frac{2}{3} = 6.5\frac{1}{3}$ lb. per sq. in. Area of piston $= 40^2 \times .7854 = 1{,}256.64$ sq. in. Total force $= 1{,}256.64 \times 6.5\frac{1}{3} = 8{,}210.05$. Adding the friction, $8{,}210.05 + 45.5 = 8{,}255.55$ lb. Ans.

(*b*) Proceeding likewise in the second case, $p\,v = p_1\,v_1$, or $p = \frac{p_1\,v_1}{v} = \frac{14.7 \times 40}{6} = 98$ lb. $98 - 14.7 = 83.3$ lb. per sq. in.

$$1{,}256.64 \times 83.3 + 45.5 = 104{,}723.612 \text{ lb. Ans.}$$

(519) 8.47 = original volume = v_1. 8.47 − 4.5 = 3.97 cu. ft. = new volume = v. By formula **53,**

$$p_1 = \frac{p\,v}{v_1} = \frac{3.97 \times 38}{8.47} = 17.812 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(520) Original weight = $W = .5$ lb. = 8 oz.; new weight = W_1 = 1 lb. 6 oz. = 22 oz. According to formula **56,** $p\,W_1 = p_1\,W$, or $p_1 = \frac{P\,W_1}{W} = \frac{14.7 \times 22}{8} = 40.425$ lb. per sq. in. Ans.

(521) Applying formula **58,**

$$v_1 = v\left(\frac{460 + t_1}{460 + t}\right) = \frac{4,516}{1,728}\left(\frac{460 + 80}{460 + 260}\right) = 1.96 \text{ cu. ft.} \quad \text{Ans.}$$

(522) According to formula **61,** $p\,V = .37052\,W\,T$, or

$$W = \frac{p\,V}{.37052\,T} = \frac{14.7 \times 1.25 \times 55}{.37052 \times 548} = 4.977 \text{ lb.} \quad \text{Ans.}$$

(523) Using formula **63,** $P\,V = p\,v + p_1\,v_1$, or $P \times 7.5 = 14.7 \times 2 \times 7.5 + 40 \times 7.5$, or $P = 69.4$ lb. per sq. in. Ans.

(524) 48″, 36″, and 24″ = 4′, 3′, and 2′, respectively. Hence, 4 × 3 × 2 = 24 cu. ft. = the volume of the block. The block will weigh as much more in a vacuum as the weight of the air it displaces. In example 510, it was found that 1 cu. ft. of air at a temperature of 60° weighed .076296 lb. .076296 × 24 + 1,200 = 1,201.83 lb. Ans.

(525) (*a*) (See Art. **1088.**) 127 + 16 = 143.

$$\frac{\left(\frac{9}{12}\right)^2 \times .7854 \times 1 \times 125 \times 62.5 \times 143}{33,000} = 14.9563 \text{ H.P.}$$

$$14.9563 \div .75 = 19.942 \text{ H.P.} \quad \text{Ans.}$$

(*b*) Discharge in gallons per hour = volume of cylinder in cu. ft. × number of strokes per minute × 7.48 × 60 = $\left(\frac{9}{12}\right)^2 \times .7854 \times 125 \times 7.48 \times 60 = 24,784.3$ gal. per hr. Ans.

(526) In this example, the number of times that the pump delivers water in 1 minute is 100 ÷ 2 = 50; in the last example, 125. Hence, the number of gallons discharged per hour in this case will be $24,786 \times \frac{50}{125} = 9,914.4$ gal. Ans.

(527) See Art. **1043.**

Pressure in condenser $= \frac{30 - 23}{30} \times 14.7 = 3.43$ lb. per sq. in. Ans.

(528) $144 \times 14.7 = 2{,}116.8$ lb. per sq. ft. Ans.

(529) $.27 \div 3 = .09$ = weight of 1 cu. ft. Using formula **56,**

$p\ W_1 = p_1\ W$, or $30\ W_1 = 65 \times .09$. $W_1 = .195$ lb. Ans.

(530) Using formula **61,**

$p\ V = .37052\ W\ T$, or $30 \times 1 = .37052 \times .09 \times T$.

$T = \frac{30}{.37052 \times .09} = 899.6°$. $899.6° - 460° = 439.6°$. Ans.

(531) $460° + 32° = 492°$; $460° + 212° = 672°$; $460° + 62° = 522°$, and $460° + (-40°) = 420°$.

(532) Using formula **61,** $p\ V = .37052\ W\ T$, and substituting,

$(14.7 \times 10) \times 4 = .37052 \times 3.5 \times T$, or $T = \frac{14.7 \times 10 \times 4}{.37052 \times 3.5} =$ $453.417°$. $453.417 - 460° = -6.583°$. Ans.

(533) Using formula **63,** $V\ P = v\ p + v_1\ p_1$, we find

$P = \frac{15 \times 63 + 19 \times 14.7 \times 3}{25} = 71.316$ lb. Ans.

(534) Using formula **60,** $p\ V = .37052\ T$, or $P = \frac{.37052 \times 540}{10} = 20$ lb. per sq. in., nearly. Ans.

(535) One inch of mercury represents a pressure of .49 lb. Therefore, the height of the mercury column is $12.5 \div .49 = 25.51$ in. Ans.

(536) Thirty inches of mercury corresponds to 34 ft. of water. (See Art. **1043.**) Therefore,

$30'' : 34$ ft. $:: 27'' : x$ ft., or $x = 30.6$ ft. Ans.

A more accurate way is $(27 \times .49) \div .434 = 30.5$ ft.

(537) (*a*) $30 - 17.5 = 12.5$ in. = original tension of gas in inches of mercury. $30 - 5 = 25$ in. = new tension in inches of mercury.

$VP = vp + v_1 p_1$ (formula **63**), or $6.7 \times 25 = 6.7 \times 12.5 + v_1 \times 30$.

$$v_1 = \frac{6.7 \times 25 - 6.7 \times 12.5}{30} = 2.79\frac{1}{6} \text{ cu. ft.} \quad \text{Ans.}$$

(*b*) To produce a vacuum of 0 inches,

$$v_1 = \frac{6.7 \times 30 - 6.7 \times 12.5}{30} = 3.908 \text{ cu. ft.} \quad \text{Ans.}$$

(538) $11 + 25 = 36$, final volume of gas. $2.4 \div 36 = \frac{1}{15}$ lb. Ans.

(539) Using formula **59**,

$$p_1 = p\left(\frac{460 + t_1}{460 + t}\right) = 12 \times \left(\frac{460 + 300}{460 + 60}\right) = 17.54 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(540) $T = 460 + 212 = 672°$. Using formula **61**, $pV = .37052\,WT$, we have $14.7 \times 1 = .37052 \times W \times 672$, or $W = \frac{14.7}{.37052 \times 672} = .059039$ lb. Ans.

(541) (*a*) $\frac{20^2 \times .7854 \times 32}{1{,}728} = 5.8178$ cu. ft. = volume of cylinder.

$32 - 26 = 6$ in., length of stroke unfinished.

$$5.8178 \times \frac{6}{32} = 1.0908 \text{ cu. ft.} \quad \text{Ans.}$$

(*b*) By formula **61**, taking the values of p, V, and T at the beginning of stroke,

$pV = .37052\ WT$, or $W = \frac{pV}{.37052\,T} = \frac{14.7 \times 5.8178}{.37052 \times 535} = .43143$ lb. Ans.

(*c*) Now, substituting in formula **61** the values of V, W, and T at time of discharge,

$$p = \frac{.37052\,WT}{V} = \frac{.37052 \times .43143 \times 585}{1.0908} = 85.727 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(542) Using formula **63**, $VP = vp + v_1p_1$, or $30 \times 35 = 19 \times 12 + 21p_1$, or $p_1 = \frac{30 \times 35 - 19 \times 12}{21} = 39.14$ lb. per sq. in. Ans.

(543) Use formula **64**. $PV = \left(\frac{p_1v_1}{T_1} + \frac{p_2v_2}{T_2}\right)T$.

$$T = 460 + 72 = 532.$$

Therefore, $P = \frac{\left(\frac{13 \times 45}{520} + \frac{17 \times 60}{540}\right)532}{60} = 26.723$ lb. per sq. in. Ans.

(544) One inch corresponds to a pressure of .49 lb. Therefore, the gauge will show $4.5 \div .49 = 9.18+$ in. See Art. **1043.**

(545) Sp. Gr. of alcohol $=.8$. Therefore, $16 \times .434 \times .8 =$ pressure exerted by the column of alcohol. $\frac{16 \times .434 \times .8}{.49} = 11.337$ in. = height of a column of mercury that will give the same pressure as 16 ft. of alcohol = number of inches shown by the gauge. Ans.

(546) (*a*) $14.7 + 9 = 23.7$ lb. per sq. in. Using formula **53**,

$$v_1 = \frac{pv}{p_1} = \frac{14.7 \times 80}{23.7} = 49.62 \text{ in.} =$$

distance between piston and end of stroke. Since the area of the piston remains constant, the volume at any point of the stroke is proportional to the distance passed over by the piston. Hence, we may use the latter for the former in the formula. $80 - 49.62 = 30.38$ in. Ans.

(*b*) Area of piston $= 80^2 \times .7854$. The volume of air at point of discharge is $80^2 \times .7854 \times 49.62$ cu. in. =

$$\frac{80^2 \times .7854 \times 49.62}{1{,}728} = 144.34 \text{ cu. ft.} \quad \text{Ans.}$$

(547) Using formula **56**, $pW_1 = p_1W$, or $3.5 \times 14.7 \times 2 = p_1 \times 13$; hence, $p_1 = \frac{14.7 \times 3.5 \times 2}{13} = 7.915+$ lb. per sq. in. Ans.

(548) $60' - 50' = 10'$. Since the volumes are proportional to the lengths of the spaces between the piston and the end of the stroke, we may apply formula **62**,

$$\frac{p\,V}{T} = \frac{p_1\,V_1}{T_1}\text{; or } \frac{14.7 \times 60}{460 + 60} = \frac{p_1 \times 10}{460 + 130}.$$

Therefore, $p_1 = \dfrac{14.7 \times 60 \times 590}{520 \times 10} = 100.07$ lb. per sq. in. Ans.

(549) $T = 127° + 460° = 587°$. Using formula **60**, $p\,V = .37052\ T$, or $V = \dfrac{.37052 \times 587}{27} = 8.055$ cu. ft. Ans.

(550) $T = 100° + 460° = 560°$.
Substituting in formula **61**, $p\,V = .37052\ W\,T$, or

$$V = \frac{.37052\ W\,T}{p} = \frac{.37052 \times .5 \times 560}{\frac{4{,}000}{144}} = 3.735 \text{ cu. ft. Ans.}$$

(551) Use formula **64.** $P\,V = \left(\dfrac{p_1\,v_1}{T_1} + \dfrac{p_2\,v_2}{T_2}\right)T$.
$T = 110° + 460° = 570°$; $T_1 = 100° + 460° = 560°$; $T_2 = 130° + 460° = 590°$.

$$\text{Therefore, } V = \frac{\left(\frac{90 \times 40}{560} + \frac{80 \times 57}{590}\right)570}{120} = 67.248 \text{ cu. ft.}$$

Ans.

(552) The pressure exerted by squeezing the bulb may be found from formula **53**, in which p is 14.7, v, the original volume = 20 cu. in., and v_1, the new volume, = 5 cu. in. $p_1 = \dfrac{p\,v}{v_1} = \dfrac{14.7 \times 20}{5} = 58.8$ lb. The pressure due to the atmosphere must be deducted, since there is an equal pressure on the outside which balances it. $58.8 - 14.7 = 44.1$ lb. per sq. in. = pressure due to squeezing the bulb. $3^2 \times .7854 = 7.0686$ sq. in. = area of bottom. $7.0686 \times 44.1 = 311.725$ lb. $7.0686 \times .434 = 3.068$ lb. = pressure due to weight of water. $311.725 + 3.068 = 314.793$ lb. Ans.

(553) Use formula **58.**

$$v_1 = v\left(\frac{460 + t_1}{460 + t}\right) = 4\left(\frac{460 + 115}{460 + 40}\right) = 4.6 \text{ cu. ft. Ans.}$$

HEAT.

(QUESTIONS 554-618.)

(554) 9 hr. 25 min. = 565 min.

55 ÷ 2 = 27.5 min. = time it would take to heat 1 lb. of the substance to this temperature under the same conditions. 27.5 ÷ 565 = .04867 = specific heat of the substance. Ans.

(555) Increase in diameter = 4.001 − 3.9985 = .0025″. $C_1 = .00000599$, from Table 19. Hence, using formula **67**, $t = \frac{1}{L\,C_1} = \frac{.0025}{3.9985 \times .00000599} = 104.4°$, nearly. Then, $104.4 + 80 = 184.4°$, nearly. Ans.

(556) (*a*) Substituting in formula **72** $W = 360$, $t_1 - t = 104.4$, and $s = .1165$; $u = W\ (t_1 - t)\ s = 360 \times 104.4 \times .1165 = 4{,}378.536$ B. T. U.

Since 12% of the total heat is lost by radiation, the above number of B. T. U. must be the number remaining after subtracting the heat units lost by radiation from the total number; or it is 100 − 12 = 88% of the total number. Hence, dividing by .88 we obtain 4,378.536 ÷ .88 = 4,975.6 B. T. U., total heat required. Ans.

(*b*) 4,975.6 × 778 = 3,871,017 ft.-lb. Ans.

(557) (*a*) $80^2 \times .7854 = 5{,}026.56$ sq. in. = area of piston.

$p\,v = p_1\,v_1$. At the beginning of the stroke $p = 14.7$, and the volume may be represented by the length of the cylinder, 80 in.; at the point of discharge p_1 is 120, therefore, $80 \times 14.7 = 120 \times v$, or $v = 9.8$ in., the portion of the stroke uncompleted at discharge.

$\frac{5{,}026.56 \times 9.8}{1{,}728} = 28.507$ cu. ft., volume of air discharged. Ans.

(b) By formula **80,** $L = 331.5744 p V \log \frac{p_1}{p} = 331.5744 \times 120 \times 28.507 \log \frac{120}{14.7} = 331.5744 \times 120 \times 28.507 \times .91186 =$ 1,034,289 ft.-lb. Ans.

(558) (a) Volume of cylinder = area of piston × length of stroke $= \frac{5{,}026.56}{144} \times \frac{80}{12} = 232.71$ cu. ft., nearly.

From formula **81,** $p_1 v_1^{1.41} = p_2 v_2^{1.41}$, or $v_2 = \sqrt[1.41]{\frac{p_1 v_1^{1.41}}{p_2}}$.

Substituting for p_1 and v_1 their values at the beginning of the stroke, and for p_2 its value at discharge, we have

$$v_2 = \sqrt[1.41]{\frac{14.7 \times 232.71^{1.41}}{120}};$$

$$\log v_2 = \frac{\log 14.7 + 1.41 \log 232.71 - \log 120}{1.41}, \text{ or}$$

$$\log v_2 = \frac{.16732 + 1.41 \times 2.36682 - 2.07918}{1.41} = 1.72011.$$

$v_2 = 52.494$ cu. ft., the volume discharged. Ans.

(b) To find the work, use formula **84,**

$$L = 351.36 p V \left[1 - \left(\frac{V}{V_1}\right)^{.41}\right] =$$

$$351.36 \times 14.7 \times 232.71 \times \left[1 - \left(\frac{232.71}{52.494}\right)^{.41}\right].$$

$$\text{Log} \left(\frac{232.71}{52.494}\right)^{.41} = .41 (\log 232.71 - \log 52.494) =$$

$.41 (2.36682 - 1.72011) = .26515$. Therefore, $\left(\frac{232.71}{52.494}\right)^{.41} =$ 1.8414.

$1 - 1.8414 = -.8414$, the minus sign indicating that the work done by the air is negative; that is, that work is done upon the air instead of by it, or that the air is compressed.

$L = 351.36 \times 14.7 \times 232.71 \times .8414 = 1{,}011{,}317$ ft.-lb. Ans.

(559) See Arts. **1090** and **1135.**

(560) Using formula **65**,

(*a*) $t_f = \frac{9}{5} \times 2{,}917 + 32 = 5{,}282.6°$ F. Ans.

(*b*) $t_f = \frac{9}{5} \times 637 + 32 = 1{,}178.6$ F. Ans.

(*c*) $t_f = \frac{9}{5} \times (-260) + 32 = -436°$ F. Ans.

(561) See Table 21, Art. **1134,** and Table 22, Art. **1141.**

$-50 - (-37.8) \times 12 \times .0333 = 4.875$ = units of heat necessary to raise the temperature to the fusion point. $5.09 \times 12 = 61.08$ B. T. U. required to melt the mercury. $[662 - (-37.8)] \times .0333 \times 12 = 279.64$ B. T. U. required to raise the temperature to the point of vaporization. $157 \times 12 = 1{,}884$ B. T. U. required for vaporizing it. $4{,}875 + 61.08 + 279.64 + 1{,}884 = 2{,}229.595$ B. T. U. Ans.

(562) Using formula **71,** $p\,V = W R\,T$, or

$$W = \frac{p v}{R T} = \frac{18 \times 100}{5.34946 \times 540} = .6231 \text{ lb. Ans.}$$

(563) See Tables 21 and 22 for specific heat and temperature of fusion of copper. 7 lb. 5 oz. = 7.3125 lb. $2{,}100° - 78° = 2{,}022°$. $2{,}022 \times 7.3125 \times .0951 = 1{,}406.13$ B. T. U. Ans.

(564) By formula **73,** $t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3} =$

$$\frac{2.25 \times .0314 \times 40 + 4 \times 65 + 1.75 \times .1298 \times 62}{2.25 \times .0314 + 4 + 1.75 \times .1298} = 64.43°. \text{ Ans.}$$

(565) Use formulas **69, 67,** and **68.** (*a*) $v = V C_3 t = 4 \times 4 \times 20 \times 1{,}200 \times .00002058 = 7.903$ cu. in. Ans.

(*b*) $l = L\,C_1 t = 20 \times 1{,}200 \times .00000686 = .16464$ in. Ans.

(*c*) $a = A\,C_2 t = 4 \times 4 \times 1{,}200 \times .00001372 = .2634$ sq. in. Ans.

(566) Using formula **71,**

$$p\,V = W R\,T, \text{ or } T = \frac{p\,V}{W R} = \frac{73 \times 61}{10 \times .38143} = 1{,}167.45°.$$

$$1{,}167.45° - 460° = 707.45°. \text{ Ans.}$$

(567) See Art. **1124.**

(568) By formula **73,** $t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3}$, or

$$t_1 = \frac{(W_1 s_1 + W_2 s_2 + W_3 s_3)\,t - W_2 s_2 t_2 - W_3 s_3 t_3}{W_1 s_1} =$$

$$\frac{(1 \times .1298 + 3.25 \times 1 + 1.5 \times .0951)\,128 - 3.25 \times 85 - 1.5 \times .0951 \times 85}{1 \times .1298} =$$

1,252°. Ans.

(569) Draw the lines $O\,X$ and $O\,Y$ (Fig. 46) perpendicular to each other. On $O\,X$ take any convenient distance, say $\frac{1}{2}$ inch, to represent one unit of volume (1 cu. ft.), and on $O\,Y$ take a distance of say 3 inches to represent the required pressure of 84 pounds. Our scale of pressure is,

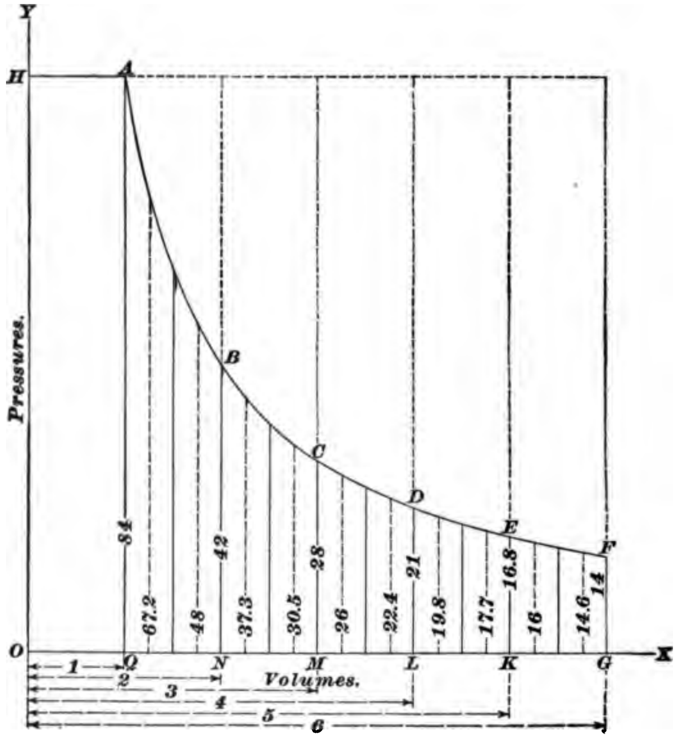

FIG. 46.

then, 28 pounds to the inch. The ordinates are now erected as shown in Art. **1156.** Thus, at A the volume is 1, and the pressure 84 pounds; hence, $p\ v = 84$. But $p\ v = p_1\ v_1 = p_2\ v_2$, etc., $= 84$; hence, $p_1 = \frac{84}{v_1}$, $p_2 = \frac{84}{v_2}$, etc. When $v = 1$, $p = \frac{84}{1} = 84 = Q\ A$.

$$\text{When } v = 2,\ p = \frac{84}{2} = 42 = N\ B.$$

$$\text{When } v = 3,\ p = \frac{84}{3} = 28 = M\ C.$$

$$\text{When } v = 4,\ p = \frac{84}{4} = 21 = L\ D.$$

$$\text{When } v = 5,\ p = \frac{84}{5} = 16.8 = K\ E.$$

$$\text{When } v = 6,\ p = \frac{84}{6} = 14 = G\ F.$$

A curve through A, B, C, D, E, and F will be the one required.

(570) Divide the space $Q\ G$ into 10 equal parts, as shown in the figure, and draw the ordinate at the middle of each space. The average ordinate will be found to be 29.95 lb. The volume represented by the length $Q\ G$ is 5 cu. ft. Hence, the work is $29.95 \times 5 \times 144 = 21{,}564$ foot-pounds. Ans.

Calculating the work by formula **79**,

$$L = 331.5744\ p\ V \log \frac{V_1}{V};$$

$$L = 331.5744 \times 84 \times 1 \times \log \frac{6}{1} = 21{,}673 \text{ ft.-lb.} \quad \text{Ans.}$$

(571) (*a*) $460 + 96 = 556°$ F. (*b*) $273\frac{1}{3} + 32 = 305\frac{1}{3}°$ C. (*c*) $273\frac{1}{3} + 180 = 453\frac{1}{3}°$ C. (*d*) $460 + 650 = 1{,}110°$ F. (*e*) $273\frac{1}{3} - 40 = 233\frac{1}{3}°$ C.

(572) $32° - 20° = 12°$. $12 \times 7 \times .504 = 42.336$ B. T. U. required to heat the ice to 32°. $142.65 \times 7 = 998.55$ B. T. U. required to melt the ice. $212° - 32° = 180°$. $180 \times 7 \times 1 = 1{,}260$ B. T. U. required to heat the *water* to 212°. $966.6 \times 7 = 6{,}766.2$ B. T. U. required to change 7 lb. of water at 212° into steam at 212°. $42.336 + 998.55 + 1{,}260 + 6{,}766.2 = 9{,}067.086$ B. T. U.

$$\frac{9{,}067.086 \times 778}{43 \times 33{,}000} = 4.971 \text{ horsepower.} \quad \text{Ans.}$$

ordinate is found to be .154; the length *a c* is 1.34; hence, area *a c b* = 1.34 × .154 = .2064 sq. in. Dividing *h g i* into 5 equal parts, the mean ordinate is found to be .82″; the length *h g* is 1.07″; hence, area *h g i* = 1.07 × .82 = .8774 sq. in. Dividing *h i b c* into 10 equal parts, the mean ordinate is found to be 1.925″; its length *h c* = 3.21″; hence, area *h i b c* = 3.21 × 1.925 = 6.179 sq. in.

1.6087 + 6.179 = 7.7877 sq in. = area *h i b c d e f A h*.

.0952 + .068 + .2064 + .8774 = 1.247 sq. in.

7.7877 − 1.247 = 6.5407 sq. in. = required area *g i b a e f g*. Ans.

(576) Using formula **73,** $t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3}$. For water, $s_1 = 1$; therefore, $W_1 t + (W_2 s_2 + W_3 s_3)\, t = W_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3$, and

$$W_1 = \frac{W_2 s_2 t_2 + W_3 s_3 t_3 - t\,(W_2 s_2 + W_3 s_3)}{t - t}.$$

Substituting the given values for W and t and the values of s_2, s_3, etc., from Table 21, we have

$$W_1 = \frac{4 \times .426 \times 80 + .5 \times .0939 \times 73 - 75.61(4 \times .426 + .5 \times .0939)}{75.61 - 73}$$

$$= \frac{7.35802}{2.61} = 2.819 \text{ lb. Ans.}$$

(577) See Arts. **1138, 1139, 1131, 1132,** and **1152.**

(578) $K = \frac{Wv^2}{2g} = \frac{120 \times 1{,}200^2}{2 \times 32.16}$ = kinetic energy in foot-pounds.

(See Art. **957.**) $\frac{120 \times 1{,}200^2}{2 \times 32.16} \times .15$ = kinetic energy expended in heat. Now, dividing by 778 to reduce the foot-pounds to B.T.U., $\frac{120 \times 1200^2 \times .15}{2 \times 32.16 \times 778} = 517.975$ B.T.U. Ans.

(579) See Art. **1126.**

(580) Using formula **73,** $t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3}$. (Since s_2, the specific heat of water, is 1, it may be left out.)

Transposing,

$$s_1 = \frac{W_2(t_2 - t) + W_3 s_3 (t_3 - t)}{W_1(t - t_1)} = \frac{1.875(91-86)+1.25 \times .0562 \times (91-86)}{5(86-40)} =$$

.0423. Ans.

(See example, Art. **1137.**)

(581) $T_1 = 460° + 450° = 910°$; $T_2 = 460° + 70° = 530°$.

Efficiency $= \frac{T_1 - T_2}{T_1} = \frac{910 - 530}{910} = 41.76\%$. Ans.

(582) Use formula **71,** $p\,V = W R\,T$. $T = 460° + 200 = 660°$; and R, from Table 20, is 5.34946; therefore, $W = \frac{p\,V}{R\,T} = \frac{20 \times 700}{5.34946 \times 660} = 3.9653$ lb. Ans.

(583) (See Arts. **1091** to **1098.**)

(*a*) $C : R = 100 : 80$, or $C = \frac{100}{80} R = \frac{5}{4} R$.

Hence, $44 \times \frac{5}{4} = 55°$ C. Ans.

(*b*) $F : R = 180 : 80$, or $F = \frac{180}{80} R = \frac{9}{4} R$.

Hence, $44 \times \frac{9}{4} + 32 = 131°$ F. Ans.

(584) By formula **81,** $p_1 v_1^{1.41} = p_2 v_2^{1.41}$. The volume of 1 lb. of air at atmospheric pressure, and having a temperature of 60°, is $\frac{1}{.076296} = 13.107$ cu. ft. (See question 510.)

Substituting in the above formula, $14.7 \times 13.107^{1.41} = 235 \times v_2^{1.41}$, or

$$v_2 = \sqrt[1.41]{\frac{14.7 \times 13.107^{1.41}}{235}} = 1.8356 \text{ cu. ft.} \quad \text{Ans.}$$

From formula **71,** remembering that $W = 1$, we have $p\,V = R\,T$, or $T = \frac{p\,V}{R} = \frac{235 \times 1.8356}{.37052} = 1{,}164.2°$. $1{,}164.2° - 460° = 704.2°$. Ans.

(585) (See Arts. **1107, 1054,** and **1099.**)

(**586**) (*a*) Cubical contents of cylinder $= (10^2 - \overline{9\tfrac{1}{4}}^2) \times .7854 \times 72 = 816.48$ cu. in. $= \frac{816.48}{1,728}$ cu. ft. Specific gravity of copper is 8.79. Then, the weight of the cylinder is $\frac{816.48}{1,728} \times 62.5 \times 8.79 = 259.58$ lb. Specific heat of copper, from Table 21, is .0951. Substituting the values of n, W, and s, in formula **72,** $n = W(t_1 - t)\,s$, we have $7,000 = 259.58\,(t_1 - t) \times .0951$, or $t_1 - t = \frac{7,000}{259.58 \times .0951} = 283.56°$ = increase of temperature.

By formula **67,** $l = L\ C_1\ t = 72 \times .00000955 \times 283.56 = .195$ in. Ans.

(*b*) By formula **69,** $v = V\ C_2\ t = 816.48 \times .00002864 \times 283.56 = 6.63$ cu. in. Ans.

(*c*) By formula **67,** $l = L\ C_1\ t = 10 \times .00000955 \times 283.56 = .027$ in. Ans.

(**587**) From Table 19, the coefficient of expansion for gases is .00203252. Substituting in formula **70,** $V_2 = \left[\frac{1 + C_3\,(t_2 - 32)}{1 + C_3\,(t_1 - 32)}\right] V_1 = \left[\frac{1 + .00203252(390 - 32)}{1 + .00203252(65 - 32)}\right] \times 12 = 19.428$ cu. ft. $19.428 - 12 = 7.428$ cu. ft. Ans.

The same answer may be obtained by using formula **58.**

(**588**) Foot-pounds per minute $= 75 \times 2 \times 20 \times 2 \times .18$.
Foot-pounds per hour $= 75 \times 2 \times 2 \times 20 \times .18 \times 60$.
Heat units per hour $= \frac{75 \times 2 \times 2 \times 20 \times .18 \times 60}{778} = 83.29$ B. T. U. Ans.

(**589**) (*a*) Using formula **87,** $T_1 = T\left(\frac{V}{V_1}\right)^{.41} = 610 \times \left(\frac{1}{4.5}\right)^{.41} = -130.76°$. Ans.

(*b*) Substituting in formula **71,** the initial values of W, R, T, and V, we obtain $p = \frac{W R T}{V} = \frac{.5 \times .37052 \times 610}{.9} = 125.565$ lb. per sq. in., the initial pressure. Ans.

(*c*) Using formula **86**,

$$p_1 = p\left(\frac{T_1}{T}\right)^{\frac{1}{.29078}} = 125.565 \times \left(\frac{329.24}{610}\right)^{\frac{1}{.29078}} = 15.06 \text{ lb. per sq. in.}$$ Ans.

(590) $12^3 \times .5236 = 904.78$ cu. in. = volume of sphere. Sp. Gr. of zinc = 6.86; therefore, $\frac{904.78}{1,728} \times 62.5 \times 6.86 =$ 224.5 lb. = weight of sphere. (See example 586.) Using formula **73**, $t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2}{W_1 s_1 + W_2 s_2} = \frac{8 \times 212 + 224.5 \times .0956 \times 70}{8 + 224.5 \times .0956}$ $= 108.49°$. $108.49° - 70° = 38.49°$ = increase in temperature of the zinc sphere after dipping in the boiling water.

Using formula **69**, $v = V C_3 t = 904.78 \times .00004903 \times 38.49 = 1.71$ cu. in., nearly. Ans.

(591) By formula **67**, $l = L\ C_1\ t$. $C_1 = .00000599$; $L = 900 \times 12$; $t = 90° - 28°$; therefore, $1 = 900 \times 12 \times .00000599 \times (90 - 28) = 4.01$ in. Ans.

(592) According to formula **72**, $n = W(t_1 - t)s$, or $s = \frac{n}{W(t_1 - t)} = \frac{n}{W} = \frac{5}{26} = .1923$. Ans.

(593) See Art. **1161**.

(594) (*a*) Temperature of vaporization of sulphur, from Table 22, is 228.3°.

Specific heat of sulphur, from Table 21, is .2026.

Latent heat of fusion of sulphur, from Table 22, is 13.26.

$228.3 - 40 = 188.3$. $13 \times .2026 \times 188.3 = 495.94454$ B. T. U. to raise sulphur to melting point. $13.26 \times 13 = 172.38$ B. T. U. to melt the sulphur. $(495.94454 + 172.38) \times 778 \times 519,956.5$ ft.-lb., total work required to perform both of the above operations. Ans.

(*b*) $\frac{519,956.5}{10 \times 33,000} = 1.5756$ horsepower. Ans.

(595) $124 \times 3 = 372$ B. T. U., heat given up by the turpentine in changing from gas to liquid. $75 \times 4 = 300$ B. T. U., heat contained in the water. $300 + 372 = 672$

B. T. U., heat contained in the mixture at the instant th turpentine has become liquid. 672 ÷ 4 = 168° = temperature of water at the instant that the turpentine has becom a liquid.

Applying formula **73,** $t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2}{W_1 s_1 + W_2 s_2} =$

$\frac{.3 \times .426 \times 313 + 168 \times 4}{3 \times .426 + 4} = 203.11°$, final temperature. An

(596) Specific heat of melted lead, from Table 21, = .040
Specific heat of solid lead, from Table 21, = .0314.
Temperature of fusion of lead, from Table 22, = 626°.
Latent heat of fusion of lead, from Table 22, = 9.67.

$$626° - 46° = 580°. \quad 800° - 626° = 174°.$$

25 × .0314 × 580 = 455.3 B. T. U., heat required to rai the lead to melting point. 9.67 × 25 = 241.75 B. T. U heat required to melt the lead. 25 × .0402 × 174 = 174. B. T. U., heat required to raise the temperature of th melted lead from fusion point to 800°. 455.3 + 241. + 174.87 = 871.92 B. T. U., total heat required. 871. × 778 = 678,353.76 ft.-lb. Ans.

(597) 32° − 10° = 22°. 22 × 10 × .504 = 110.88 B. T. U heat required to raise ice from 10° to 32°. Latent heat fusion of ice, from Table 22, is 142.65; then 142.65 × = 1,426.5 B. T. U., heat units to melt the ice. 1,426 + 110.88 = 1,537.38 B. T. U., total heat units to be tak from the mixture to melt the ice. Hence, applying formu **73,**

$$t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 t_3 + W_4 t_4 + W_5 s_5 t_5 - 1{,}537.38}{W_1 s_1 + W_2 s_2 + W_3 + W_4 + W_5 s_5} =$$

$$\frac{11.5 \times .1298 \times 180 + 42 \times .0939 \times 240 + 10 \times 32 + 50 \times 120 + 20 \times .0314 \times 80 - 1{,}537.38}{11.5 \times .1298 + 42 \times .0939 + 10 + 50 + 20 \times .0314}$$

$= 91.55°$. Ans.

(598) (*a*) $p_1 v_1^{1.41} = p_2 v_2^{1.41}$, or $p_2 = \frac{140 \times 3^{1.41}}{16^{1.41}} = 13.21$

$$\text{Area} = \frac{p_1 v_1 - p_2 v_2}{.41} = \frac{140 \times 3 - 13.215 \times 16}{.41} = 508.69.$$

$$508.69 \div 20 = 25.434 \text{ sq. in.} \quad \text{Ans.}$$

(*b*) $25.434 \div 13 = 1.9565$. Ans.

(*c*) $1.9565 \times 20 = 39.13$ lb. per sq. in. Ans.

(599) Use formula **66,** $t_c = (t_f - 32)\frac{5}{9}$.

(*a*) $t_c = (-10 - 32) \times \frac{5}{9} = -23\frac{1}{3}^\circ$ C. Ans.

(*b*) $t_c = (25 - 32) \times \frac{5}{9} = -3\frac{8}{9}^\circ$ C. Ans.

(*c*) $t_c = (2{,}200 - 32) \times \frac{5}{9} = 1{,}204\frac{4}{9}^\circ$ C. Ans.

(600) This must be answered by the student.

(601) Using formula **80,** $L = 331.5744\, p\, v \log \frac{p_1}{p} =$ $331.5744 \times 15 \times 10 \times \log \frac{85}{15} = 37{,}467.74$ ft.-lb. Ans.

(602) 520 H. P. = $520 \times 33{,}000$ ft.-lb. per minute = $520 \times 33{,}000 \times 60$ ft.-lb. per hour = $\frac{520 \times 33{,}000 \times 60}{778}$ B. T. U. = 1,323,393.31 B. T. U. Ans

(603) Let l = the required latent heat;

W_1 = given weight of zinc;
W_2 = given weight of water;
s_1 = specific heat of zinc;
s_2 = specific heat of water (= 1);
t_1 = temperature of melting zinc;
t_2 = temperature of water.

Then, the total heat of the mixture is $W_1 s_1 t_1 + W_2 s_2 t_2 + W_1 l$, and from formula **73,** the temperature

$$t = \frac{W_1 s_1 t_1 + W_2 t_2 + W_1 l}{W_1 s_1 + W_2}, \text{ or}$$

$$l = \frac{t(W_1 s_1 + W_2) - W_1 s_1 t_1 - W_2 t_2}{W_1} =$$

$$\frac{102\frac{1}{3}(4 \times .0956 + 10) - 4 \times .0956 \times 680 - 10 \times 60}{4} = 50.61.$$ Ans.

(604) See Arts. **1117** and **1120.**

(605) (*a*) and (*b*) See Art. **1118.**

(*c*) A non-conductor is one which will not conduct heat. There really is no such substance, but some substances are such bad conductors that they are termed non-conductors.

(606) When answering this question, consult Art. **1123.**

(607) In Art. **1130** it is stated that 1 calorie = 3.96 B. T. U. Hence, (*a*) 798 B. T. U. = 798 ÷ 3.96 = 201.515 calories. Ans.

(*b*) 40 × 3.96 = 158.4 B. T. U. Ans.

(608) (*a*) First calculate the weight by formula **71.**

$$W = \frac{pV}{RT} = \frac{18 \times 7.68}{.33552 \times 500} = .824 \text{ lb., nearly.}$$

Units of heat required $= s_v\, W\,(T_1 - T) = s_v\, W(t_1 - t)$ $= .15507 \times .824 \times (416 - 40) = 48.0444$ B. T. U.

Therefore, 48.0444 × 778 = 37,379 ft.-lb., nearly. Ans.

(*b*) .21751 × .824 (416 – 40) × 778 = 52,429 ft.-lb., nearly. Ans.

(609) See Arts. **1144** to **1147.**

(610) See Arts. **1155** and **1164.**

(611) Using formula **87,**

$$T_1 = T\left(\frac{V}{V_1}\right)^{.41} = 528\left(\frac{3.72}{1.2}\right)^{.41} = 839.64° = 380°, \text{ nearly, above}$$

0° F. Ans.

(612) (See Arts. **1149** and **1177.**)

(613) Use formula **81,** $p\,v^{1.41} = p_1\,v_1^{1.41}$.

$$14.7 \times 48^{1.41} = p_1 \times (48 - 38)^{1.41}, \text{ or}$$

$$p_1 = 14.7\left(\frac{48}{10}\right)^{1.41} = 134.24 \text{ lb. per sq. in. Ans.}$$

Now, using formula **86,**

$$T_1 = 500\left(\frac{134.24}{14.7}\right)^{.29078} = 951°, \text{ nearly, } = 491° \text{ above } 0° \text{ F.}$$

Ans.

STEAM AND STEAM ENGINES.

(QUESTIONS 614–688.)

(614) See Arts. **1191** and **1192.**

(615) See Arts. **1189** to **1195.**

(616) See Arts. **1198** to **1201.**

(617) Log $p = 6.1007 - \frac{2,719.78}{T} - \frac{400,215}{T^2} = 6.1007 - \frac{2,719.78}{290 + 460} - \frac{400,215}{(290 + 460)^2} = 1.76284$. Hence, $p = 57.92$ lb. per sq. in. Ans.

(618) Using formula **91,**

$H = 1,081.94 + 305\,t = 1,081.94 + .305 \times 318 = 1,178.93$ B. T. U. Ans.

(619) See Arts. **1217** and **1218.**

(620) Using formula **92,**

$$p\,v^{\frac{17}{16}} = 475, \text{ or } v^{\frac{17}{16}} = \frac{475}{p} = \frac{475}{81}.$$

Hence,

$$\tfrac{17}{16} \log v = \log 475 - \log 81 = 2.67669 - 1.90849 = .76820,$$

$$\log v = .76820 \times \frac{16}{17} = .72301.$$

$$v = 5.2846 \text{ cu. ft.} = \text{volume of one pound.}$$

$$5.2846 \times 6 = 31.7076 \text{ cu. ft.} \quad \text{Ans.}$$

(621) Using formula **92,**

$$p\,v^{\frac{17}{16}} = 475, \text{ or } p = \frac{475}{v^{\frac{17}{16}}} = \frac{475}{7^{\frac{17}{16}}}.$$

$$\text{Hence, } \log p = \log 475 - \frac{17}{16} \log 7 = 1.77877.$$

$$p = 60.086 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(622) (a) Looking in the table of Properties of Saturated Steam, it is found that for a temperature of 320.094° the pressure is 90 pounds, and for the next lower pressure of the table, 88 pounds, the temperature is 318.510°. Hence, the difference in temperature for a difference of one pound in pressure is

$$\frac{320.094 - 318.510}{2} = .792°.$$

$$320.094° - 320° = .094° = \text{actual difference.}$$

Then 1 lb. : .792° :: x lb. : .094, or $x = .12$ lb., nearly = difference in pressure between 320° and 320.094°.

$90 - .12 = 89.88$ lb. per sq. in., absolute pressure.

$89.88 - 14.7 = 75.18$ lb. per sq. in., gauge pressure. Ans.

(b) 110 lb. gauge pressure $= 110 + 14.7 = 124.7$ lb., absolute.

Temperature corresponding to 125 lb. = 344.136°.

Temperature corresponding to 120 lb. = 341.058°.

Difference, per pound difference in pressure =

$$\frac{344.136° - 341.058°}{5} = .616°.$$

$125 - 124.7 = .3$ lb., difference in pressure; $.616 \times .3 = .185°$.

$$344.136 - .185 = 343.951°. \quad \text{Ans.}$$

(c) 70 lb., gauge pressure, $= 70 + 14.7 = 84.7$ lb., absolute.

Heat of liquid for 84 lb. = 285.414 B. T. U.

Heat of liquid for 86 lb. = 287.096 B. T. U.

$$\text{Difference per pound} = \frac{287.096 - 285.414}{2} = .841 \text{ B. T. U.}$$

$.841 \times .7 = .589$. $285.414 + .589 = 286.003$. B. T. U. Ans.

Total heat at 86 lb. = 1,178.592 B. T. U.

Total heat at 84 lb. = 1,178.091 B. T. U.

Difference per pound $= \frac{1{,}178.592 - 1{,}178.091}{2} = .25$ B. T. U.

$.25 \times .7 = .175$. $1{,}178.091 + .175 = 1{,}178.266$ B. T. U. Ans.

(*d*) Latent heat per pound at 66 lb. pressure = 904.443 B. T. U.

Latent heat per pound at 68 lb. pressure = 903.02 B. T. U.

Difference per pound $= \frac{904.443 - 903.02}{2} = .712$ B. T. U.

$904.443 - .712 = 903.731$ B. T. U., latent heat per pound at 67 lb. pressure.

$903.731 \times 3 = 2{,}711.193$ B. T. U. Ans.

(623) (*a*) The isothermal of saturated steam is a straight line parallel to the axis of volumes.

(*b*) The equilateral hyperbola.

(624) From the table of the Properties of Saturated Steam, the weight of one cubic foot of steam at a pressure of 44 lb. is .106345 lb., and at a pressure of 42 lb. it is .101794 lb.

Difference per pound $= \frac{.106345 - .101794}{2} = .002276$ lb.

The weight of a cubic foot at 43 lb. pressure is

$.101794 + .002276 = .10407$ lb.

Weight of 38 cu. ft. $= .10407 \times 38 = 3.95466$ lb. Ans.

(625) See Arts. **1214** and **1215.**

(626) See Art. **1206.**

To raise 1 pound of water from 55° to 140° requires $140 - 55 = 85$ B. T. U. Then, to raise 300 pounds of water through the same range of temperature requires $85 \times 300 = 25,500$ B. T. U.

60 pounds gauge pressure $= 60 + 14.7 = 74.7$ pounds, absolute.

Total heat of 1 pound of steam at 74 pounds pressure $= 1,175.431$ B. T. U.

Total heat of 1 pound of steam at 76 pounds pressure, $= 1,175.985$ B. T. U.

Difference per pound pressure $= \frac{1,175.985 - 1,175.431}{2} =$ .277 B. T. U.

$.277 \times .7 = .194$. $1,175.431 + .194 = 1,175.625$ B. T. U., total heat of 1 pound of steam at an absolute pressure of 74.7 pounds per sq. in.

The total heat (above 32°) of a pound of water at 140° is $140 - 32 = 108$ B. T. U.

Hence, each pound of steam on being mixed with water lowers in temperature to 140°, and in so doing gives up $1,175.625 - 108 = 1,067.625$ B. T. U.

The required amount of steam is

$$\frac{25,500}{1,067.625} = 23.885 \text{ lb.} \quad \text{Ans.}$$

(627) The temperature 256° does not appear in the table of the Properties of Saturated Steam. The next lower temperature is found to be 254.002°; and the next higher, 257.523°. The difference is $257.523 - 254.002 = 3.521°$. The difference between the lower temperature and 256° is $256 - 254.002 = 1.998°$. The volume of a pound of steam at the lower temperature is 12.68 cu. ft.; and at the higher temperature, 11.98 cu. ft. The difference is $12.68 - 11.98 = .7$ cu. ft.

The difference between the volume at the lower temperature and the volume at 256° may now be found by proportion

$$3.521 : 1.998 :: .7 : x, \text{ or } x = .397.$$

Hence, the required volume of one pound of steam at 256° is

$$12.68 - .397 = 12.283 \text{ cu. ft. Ans.}$$

$$12.283 \times 4\frac{1}{3} = 53.226 \text{ cu. ft. Ans.}$$

(628) See Art. **1192.** Specific heat of superheated steam is .48. Hence, to raise 1 pound of superheated steam from 310° to 342° requires

$$.48 \times (342 - 310) = 15.36 \text{ B. T. U.}$$

To raise 6 pounds through the same range of temperature requires

$$15.36 \times 6 = 92.16 \text{ B. T. U. Ans.}$$

(629) (*a*) $\log p = 6.1007 - \frac{2{,}719.78}{T} - \frac{400{,}215}{T^2}$

$$= 6.1007 - \frac{2{,}719.78}{254 + 460} - \frac{400{,}215}{(254 + 460)^2}$$

$$= 1.50643.$$

$$p = 32.094 \text{ lb. per sq. in. Ans.}$$

From the tables, $p = 32$ lb. per sq. in. for a temperature of 254°.

(*b*) $\log p = 6.1007 - \frac{2{,}719.78}{377 + 460} - \frac{400{,}215}{(377 + 460)^2} = 2.27999,$

or $p = 190.54$ lb. per sq. in. Ans.

From the tables, $p = 189.212$ lb. per sq. in. for a temperature of 377°.

(630) See Arts. **1220** and **1221.**

(631) (*a*) $H = 1{,}081.94 + .305\,t$. From the table, $t = 385.759°$.

Hence, $H = 1{,}081.94 + .305 \times 385.759 = 1{,}199.596$ B. T. U. Ans.

From the tables, $H = 1{,}199.597$.

(*b*) $p = 88$; $t = 318.51°$.

Hence, $H = 1{,}081.94 + .305 \times 318.51 = 1{,}179.086$ B. T. U. Ans.

From the tables, $H = 1{,}179.085$.

(*c*) $p = 37$; $t = 260.883 + \frac{264.093 - 260.883}{2} = 262.488°$.

$H = 1{,}081.94 + .305 \times 262.488 = 1{,}161.999$ B.T.U. Ans.

From the tables, $H = 1{,}161.999$.

(632) See Art. **1293.**

(633) See Fig. 49. The lines $O\,X$ and $O\,Y$ are drawn at right angles. The scale of volumes chosen is 20 cu. ft. to the inch, and the scale of pressures 20 lb. to the inch. The volumes corresponding to 50, 40, 30 lb., etc., are found from the steam table, and laid off along $O\,X$. The pressures are then laid off vertically, as shown, forming the required curve $A\,B$.

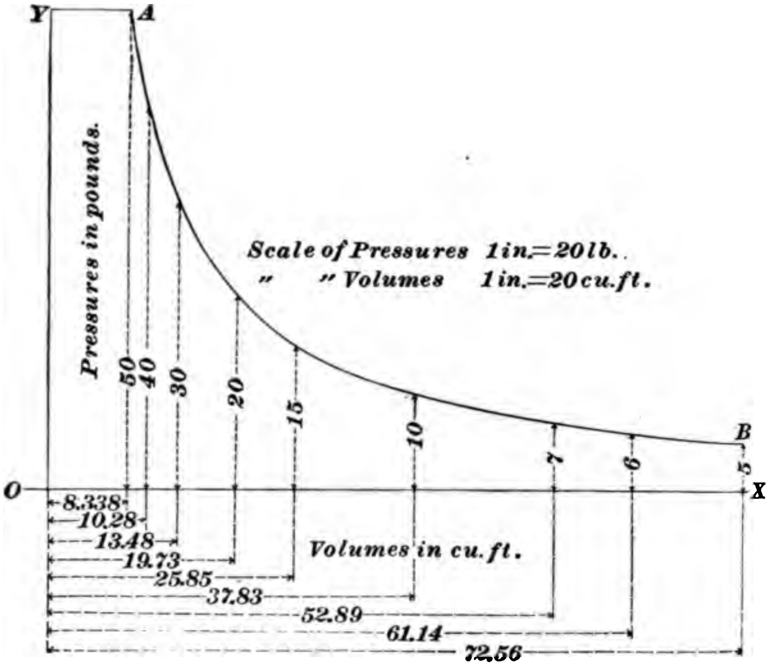

FIG. 49.

(634) See Art. **1224.**

(635) (*a*) See Arts. **1252** to **1254.**

(*b*) Clearance $i = .07$; apparent cut-off $k_1 = \frac{3}{8} = .375$. Then, by formula **97,** the real cut-off =

$$k = \frac{k_1 + i}{1 + i} = \frac{.375 + .07}{1 + .07} = \frac{.445}{1.07} = .416. \quad \text{Ans.}$$

By formula **96,** ratio of expansion, $e = \frac{1}{k} = \frac{1}{.416} = 2.4.$ Ans.

(636) By formula **97,** $k = \frac{k_1 + i}{1 + i} = \frac{.625 + .06}{1 + .06} = \frac{.685}{1.06}.$

By formula **96,** $e = \frac{1}{k} = \frac{1}{\frac{.685}{1.06}} = \frac{1.06}{.685} = 1.547.$

By formula **99,** M. E. P. $= \frac{.9\,P(1 + 2.3 \log e)}{e} - .9p.$

$$P = 60 + 14.7 = 74.7 \text{ lb.};\ p = 17 \text{ lb.}$$

Then, M. E. P. $= \frac{.9 \times 74.7\,(1 + 2.3 \log 1.547)}{1.547} - .9 \times 17 =$ 47.1 lb. per sq. in. Ans.

(637) (*a*) Using formula **98,**

I. H. P. $= \frac{PLAN}{33{,}000} = \frac{47.1 \times \frac{18}{12} \times 11^2 \times .7854 \times 210 \times 2}{33{,}000} =$ 85.45 H. P. Ans.

(*b*) 85.45 × .83 = 70.92 actual H. P. Ans.

(*c*) 85.45 − 70.92 = 14.53 friction H. P. Ans.

(638) See Arts. **1232, 1236, 1244,** and **1245.**

(639) (*a*) See Arts. **1262** and **1276.**

(*b*) The three principal uses are the following:

1. To find the I. H. P. of the engine.
2. To detect defects in valve setting, and to serve as a guide in setting the valves.
3. To determine, approximately, the steam consumption of the engine.

(640) (*a*) Work per stroke per sq. in. of piston =

$$3.47 \times \frac{6}{12} \times 40 = 69.4 \text{ ft.-lb.}$$

Area of piston $= 16^2 \times .7854 = 201.0624$ sq. in.

Then, total work per stroke $= 69.4 \times 201.0624 = 13{,}953.73$ ft.-lb. Ans.

(*b*) 120 rev. per min. $= 120 \times 2 = 240$ strokes per min.

$$\text{Then, H. P.} = \frac{13{,}953.73 \times 240}{33{,}000} = 101.48. \quad \text{Ans.}$$

(641) Admission, cut-off, release, and compression are all too late, and the back pressure is excessive. Admission, release, and compression may be made earlier by increasing the angular advance; cut-off may be made earlier by adding lap to the valve; the back pressure can be reduced only by reducing the resistance to the passage of the exhaust steam; that is, by enlarging the exhaust port or the exhaust pipe, or both.

(642) Assuming a mechanical efficiency of 80%, the I. H. P. of the engine must be $\frac{120}{.80} = 150$.

The probable M. E. P. may be found from formula **99.**

$$\text{M. E. P.} = \frac{.9\,P(1 + 2.3 \log e)}{e} - .9\,p =$$

$$\frac{.9 \times 84.7\,(1 + 2.3 \log 4)}{4} - .9 \times 3 = 42.75 \text{ lb. per sq. in.}$$

Assume 550 feet per minute as a fair piston speed. We have, then, $P = 42.75$, and $L\,N$ = piston speed = 550. Substituting, in formula **98,** I. H. P. $= \frac{P\,L\,A\,N}{33{,}000}$, $150 = \frac{42.75 \times 550 \times A}{33{,}000}$, or $A = \frac{150 \times 33{,}000}{42.75 \times 550} = 210.52$ sq. in., the area of the piston. The diameter is, consequently, $\sqrt{\frac{210.52}{.7854}} = 16\frac{3}{8}''$, nearly. Assuming the stroke to be 42 in., the number of revolutions, from formula **100,** is

$$R = \frac{6\,S}{L} = \frac{6 \times 550}{42} = 78.57.$$

This is a fair number of revolutions for a Corliss engine. Hence, an engine $16\frac{1}{2}'' \times 42''$, making $78\frac{1}{2}$ rev. per min., will do the required work.

(643) (*a*) 60 pounds, gauge pressure, $= 60 + 14.7 = 74.7$ lb., absolute; 2 lb. above atmosphere $= 2 + 14.7 = 16.7$ lb., absolute.

Temperature of steam at 74.7 lb. pressure is, from the table of the Properties of Saturated Steam,

$$306.526 + \left(\frac{308.344 - 306.526}{2}\right).7 = 307.162^\circ.$$

Temperature of steam at 16.7 is $216.347 + (219.452 - 216.347).7 = 218.521^\circ$. Then, $T_1 = 307.162^\circ + 460 = 767.162$ and $T_2 = 218.521 + 460 = 678.521^\circ$.

Thermal efficiency $= \frac{T_1 - T_2}{T_1} = \frac{767.162 - 678.521}{767.162} = .1155 = 11.55\%$. Ans.

(*b*) In this case, the absolute pressure of the entering steam is $90 + 14.7 = 104.7$ pounds per sq. in., and the pressure of the exhaust steam 3 pounds per sq. in., absolute.

The temperature corresponding to the former pressure is from the table $= 331.169 - \left(\frac{331.169 - 327.625}{5}\right).3 = 330.956^\circ$. The temperature corresponding to the latter pressure is 141.654°. Hence, T_1 is $330.956 + 460 = 790.956^\circ$, and T_2 is $141.654 + 460 = 601.654^\circ$.

Thermal efficiency $= \frac{790.956 - 601.654}{790.956} = .2393 = 23.93\%$. Ans.

(644) (*a*) Using formula **100**,

$$L = \frac{6\,S}{R} = \frac{6 \times 540}{150} = 21.6''. \quad \text{Ans.}$$

$$(b)\ R = \frac{6\,S}{L} = \frac{6 \times 900}{2\frac{1}{2} \times 12} = 180 \text{ rev. per min.} \quad \text{Ans.}$$

(645) The M. E. P. is found as shown in Fig. 50.

The middle ordinates are drawn as explained in Art. **1159.**

These ordinates are measured and multiplied by the scale of the spring, which reduces them from inches to pounds.

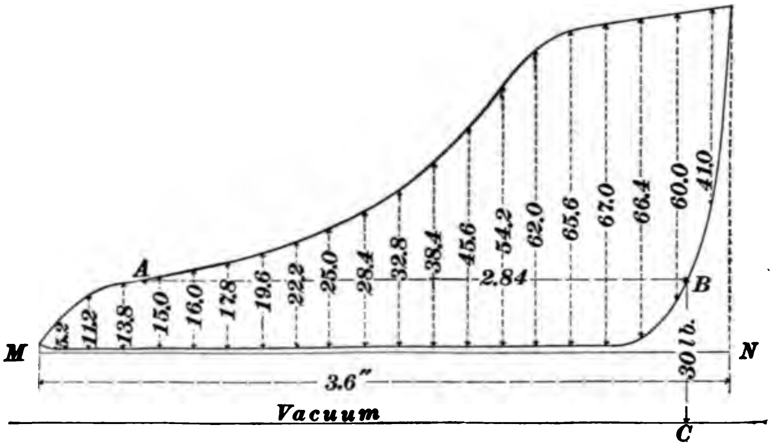

FIG. 50.

In the figure the sum of the 20 ordinates is 707.2 pounds. The mean ordinate, or, what is the same thing, the M. E. P., is, therefore, $\frac{707.2}{20} = 35.36$ lb. Draw the vacuum line at a distance of $\frac{14.7}{40} = .3675''$ below the atmospheric line MN. Choose the point A near the point of release, and draw AB parallel to MN. The height BC of this line above the vacuum line is $\frac{3''}{4}$; therefore, the absolute pressure at A or B is $\frac{3}{4} \times 40 = 30$ pounds. The length $AB = l = 2.84''$, and the length of the diagram L is $3.6''$. The weight of a cu. ft. of steam at 30 pounds pressure, absolute, is .074201 lb.

Substituting these values in formula **101,**

$$Q = \frac{13{,}750\, l\, W}{P L} = \frac{13{,}750 \times 2.84 \times .074201}{35.36 \times 3.6} =$$

22.76 lb. per I. H. P. per hour. Ans.

(646) (*a*) See Arts. **1293** and **1294.**

(*b*) See Arts. **1314** and **1315.**

(647) Area of high-pressure cylinder $= 19^2 \times .7854$ sq. in. Area of low-pressure cylinder $= 32^2 \times .7854 = 804.25$ sq. in. Ratio of area of high-pressure cylinder to area of low-pressure cylinder $= \frac{19^2 \times .7854}{32^2 \times .7854} = \frac{19^2}{32^2} = \frac{361}{1{,}024}$.

M. E. P. of high-pressure cylinder reduced to low-pressure cylinder $= 52 \times \frac{361}{1{,}024} = 18.332$ lb. per sq. in.

Total M. E. P. reduced to low-pressure cylinder $=$
$18 + 18.332 = 36.332$ lb. per sq. in.

(*a*) Substituting now in formula **98,**

$$\text{I. H. P.} = \frac{P L A N}{33{,}000} = \frac{36.332 \times \frac{30}{12} \times 804.25 \times 120 \times 2}{33{,}000} = 531.29.$$ Ans.

(*b*) Since the stroke is the same for each cylinder, the ratio of the work done by the two cylinders is proportional to the ratio of the two M. E. P.'s reduced to the low-pressure cylinder. Hence, the ratio of the work done in the high-pressure cylinder to that done in the low-pressure cylinder is $18.332 : 18 = 1.0184$. Ans.

(648) (*a*) See Art. **1276.**

(*b*) 1. Decrease the angular advance.

2. Increase the angular advance.

3. Lower the boiler pressure or decrease the number of revolutions.

4. Raise the boiler pressure or increase the number of revolutions.

(649) (*a*) The points desired are shown in Fig. 51. To locate the point of cut-off, prolong the steam and expansion

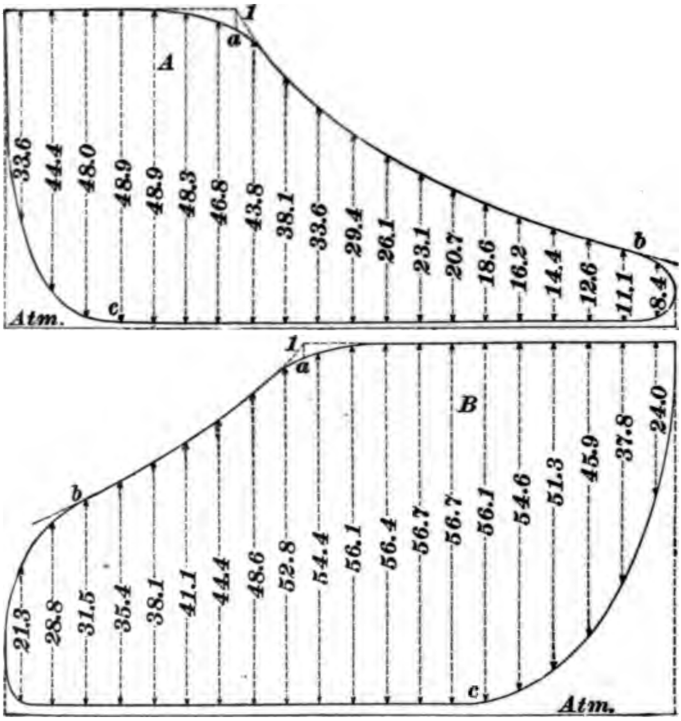

FIG. 51.

lines till they intersect, as at *1*. From *1* drop the perpendicular *1 a*, and *a* will be the point of cut-off.

The point of release may be located by prolonging the expansion curve, and noting the point where the actual curve departs from it, as shown at *b*. The point of compression *c* is easily located.

(*b*) The M. E. P. of the two diagrams are found as shown in the figure. The length of each diagram is divided into 20 equal parts, and ordinates are erected at the middle points of the divisions. The lengths of these ordinates multiplied by the scale of the spring used, in this case, 1″ = 30 lb., added together and divided by 20 gives the required M. E. P. The

M. E. P. of diagram A is found to be 30.75 lb. per sq. in., and the M. E. P. of diagram B is found to be 44.6 lb. per sq. in. Ans.

(650) Using formula **106,**

$S = .0944\, W = .0944 \times 675 \times 26\frac{1}{2} = 1{,}688.58$ sq. ft. Ans.

(651) Area of piston $= .7854 \times 30^2 = 706.86$ sq. in. The piston speed is, from formula **100,** $S = \frac{L\,R}{6} = \frac{48 \times 80}{6} =$ 640 ft. per min.; hence, the velocity of the crank-pin ($= v_0$) is $\frac{640 \times 3.1416}{2} = 1{,}005.3$ ft. per min. $= \frac{1{,}005.3}{60} = 16.755$ ft. per sec. The ratio n between the radius of the fly-wheel and length of crank is $\frac{10}{2} = 5$. Substituting now in formula **107,**

$$W = \frac{A\,H\,g}{n^2\,E\,V_0^{\,2}} = \frac{706.86 \times 88 \times 32.16}{5^2 \times \frac{1}{50} \times 16.755^2} = 14{,}251.92 \text{ lb. Ans.}$$

(652) (*a*) Use formula **98.**

$$\text{I. H. P.} = \frac{P\,L\,A\,N}{33{,}000}, \text{ or } A = \frac{33{,}000 \text{ I. H. P.}}{P\,L\,N} = \frac{33{,}000 \times 1{,}200}{42 \times \frac{42}{12} \times 70 \times 2} =$$

1,924.2 sq. in., area of low-pressure piston. Diameter of low-pressure cylinder $= \sqrt{\frac{1{,}924.2}{.7854}} = 49\frac{1}{2}''$. Ans.

(*b*) Since the stroke is the same for both cylinders, their volumes are proportional to the areas of their pistons.

Hence, using formula **103,**

$\frac{V}{v} = \frac{\text{area of low-pressure piston}}{\text{area of high-pressure piston}} = \frac{E}{2.72}$, or area of high-pressure piston $= \frac{2.72 \times 1{,}924.2}{5.5} = 951.6$ sq. in.

The corresponding diameter is

$$\sqrt{\frac{951.6}{.7854}} = 34\frac{13}{16}'', \text{ or say } 34\frac{7}{8}''. \text{ Ans.}$$

Use formula **104,**

$$\frac{V}{v} = \frac{\text{area of low-pressure piston}}{\text{area of high-pressure piston}} = \sqrt{E} = \sqrt{5.5}.$$

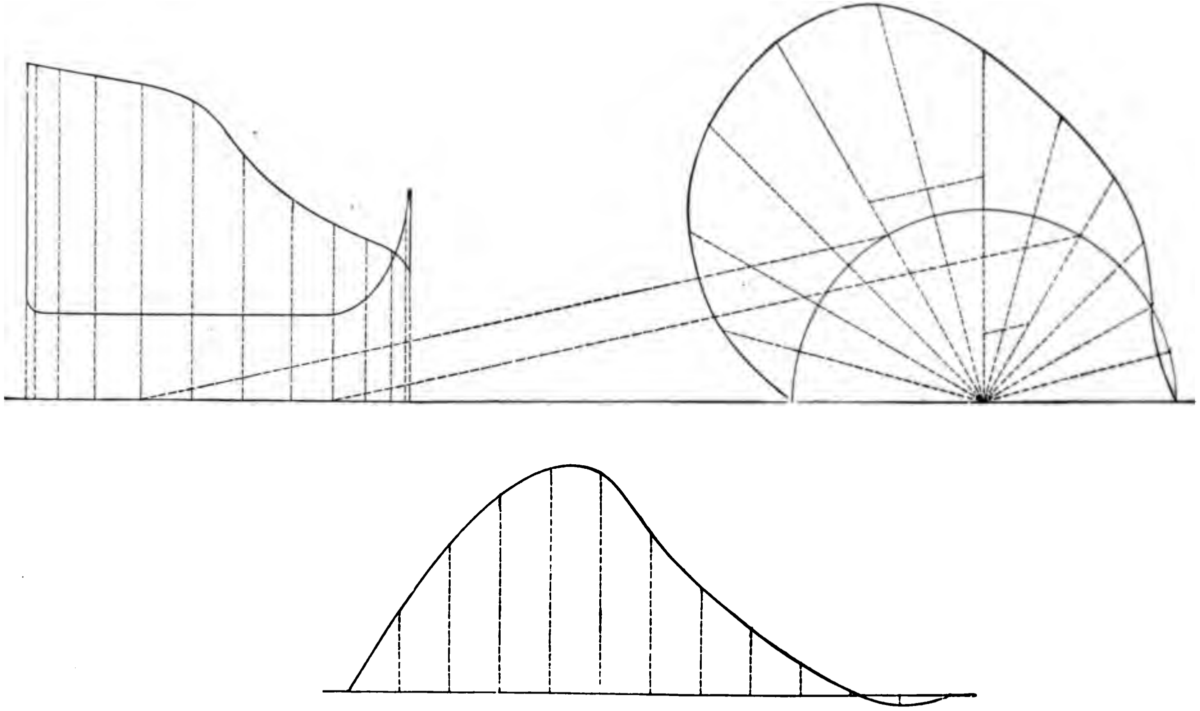

FIG. 52.

Hence, area of high-pressure piston $=\frac{1,924.2}{\sqrt{5.5}}=820.48$ sq. in.

The corresponding diameter is $32\frac{5''}{16}$, say $32\frac{3''}{8}$. Ans.

(653) (*a*) By formula **102,** $E=\frac{eV}{v}$, or $e=\frac{Ev}{V}$; but, by formula **103,** $\frac{V}{v}=\frac{E}{2.72}$, or $E=2.72\frac{V}{v}$. Substituting this value of E in **102,** we have, $e=2.72\frac{V}{v}\,\frac{v}{V}=2.72$. Hence, the real cut-off $=\frac{1}{e}=\frac{1}{2.72}=.368$. Ans.

(*b*) From formula **104,** $\frac{V}{v}=\sqrt{E}$, or $\frac{v}{V}=\frac{1}{\sqrt{E}}$.

Substituting this value of $\frac{v}{V}$ in formula **102,** above,

$$e=\frac{E}{\sqrt{E}}=\sqrt{E}=\sqrt{5.5}=2.345.$$

Real cut-off $=\frac{1}{e}=\frac{1}{2.345}=.426$. Ans.

(654) See Art. **1330.**

(655) The construction is shown in Fig. 52. It is precisely similar to the construction shown in Art. **1313,** and need not be further described.

(656) The total heat of steam at $4\frac{1}{2}$ pounds pressure is, from the steam tables,

$$\frac{1,128.641+1,131.462}{2}=1,130.051 \text{ B. T. U.}$$

Now, using formula **105,**

$$W=\frac{H-t_1+32}{t_1-t_2}=\frac{1,130.051-130+32}{130-55}=13.76 \text{ lb. Ans.}$$

(657) See Art. **1291.**

(658) (*a*) Using formula **98,** I. H. P. $=\frac{PLAN}{33,000}$, or

$$LA=\frac{33,000 \text{ I. H. P.}}{PN}=\frac{33,000\times 42}{36.3\times 155\times 2}.$$

L = stroke in feet.

$12\,L$ = stroke in inches.

From the conditions of the problem, $12\,L = 1\frac{1}{3}\,d$, when d is taken in inches, or $L = \frac{1}{9}\,d$. $A = .7854\,d^2$. Substituting these values of L and A,

$$\frac{1}{9}\,d \times .7854\,d^2 = \frac{.7854\,d^3}{9} = \frac{33{,}000 \times 42}{2 \times 36.3 \times 155},$$

or $d = \sqrt[3]{\dfrac{33{,}000 \times 42 \times 9}{36.3 \times 155 \times 2 \times .7854}} = 11.22''$ nearly, or, say $11\frac{1}{4}''$. Ans.

Stroke $= 1\frac{1}{3}\,d = 11\frac{1}{4} \times 1\frac{1}{3} = 15''$, nearly. Ans.

(*c*) Using formula **100**,

$$S = \frac{L\,R}{6} = \frac{15 \times 155}{6} = 387.5 \text{ ft. per min.} \quad \text{Ans.}$$

(659) (*a*) Area of piston, $.7854 \times 22^2 = 380.1336$ sq. in.

Total pressure on piston $= 380.1336 \times 72 = 27{,}369.62$ lb.

At the beginning of the stroke, the piston rod, connecting-rod, and crank lie in the same straight line, and, consequently, the total pressure on the piston is transmitted to the crank-shaft.

(*b*) In this case the total pressure on the piston is $380.1336 \times 65 = 24{,}708.68$ lb., which is also the horizontal pressure on the crank-pin.

Then, the pressure on the crank-shaft is $24{,}708.68 \times \cos 60^\circ = 12{,}354.34$ lb. Ans.

(*c*) The tangential pressure is $24{,}708.68 \times \sin 60^\circ = 24{,}708.68 \times .86603 = 21{,}398.46$ lb. Ans.

(660) See Art. **1311.**

The mean ordinate should be $34.6 \times \frac{2}{3.1416} = 22.03$ lb. per sq. in. Ans.

(661) Diameter of drivers $= 80'' = \frac{80}{12} = 6\frac{2}{3}$ ft.

Circumference of drivers $= 6\frac{2}{3} \times \pi = 20.944$ ft.

60 miles per hour $=$ 1 mile per min.

One mile contains 5,280 ft. Hence, the number of revolutions of the driver per min. is $\frac{5,280}{20.944}$.

Area of piston $= .7854 \times 19^2 = 283.53$.

Now, using formula **98**,

$$\text{I.H.P.} = \frac{PLAN}{33,000} = \frac{52.5 \times \frac{24}{12} \times 283.53 \times \frac{5,280}{20.944} \times 2}{33,000} = 454.86.$$

Since there are two cylinders, the total horsepower $=$ $454.86 \times 2 = 909.72$ I. H. P. Ans.

(662) (*a*) As shown in Fig. 53, the diameter of the path described by the eccentric is the travel of the valve, 5″.

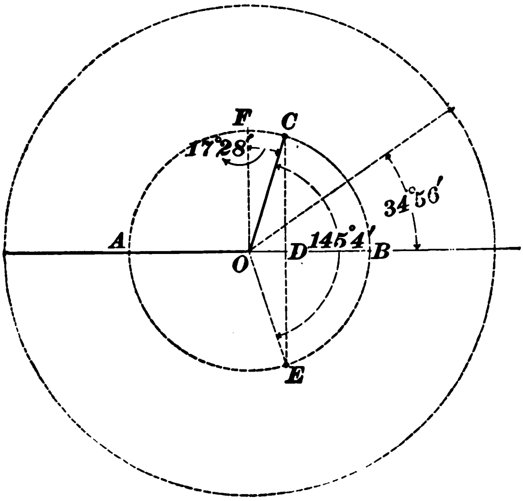

FIG. 53.

The eccentric crank must be in a position *O C*, so that, upon dropping the perpendicular $C\,D$ upon $A\,B$, the distance $O\,D$ will equal the lap $\frac{3''}{4}$. Then, $\cos C\,O\,D = \frac{O\,D}{O\,C} = \frac{\frac{3}{4}}{2\frac{1}{2}} = .3$.

The angle whose cosine is .3 is 72° 32′, nearly. Hence, $C\,O\,D = 72° \; 32'$, and the angular advance $= C\,O\,F = 90° - C\,O\,D = 90 - 72° \; 32' = 17° \; 28'$.

(*b*) When cut-off occurs the valve must be in the same position as it was at the beginning of the stroke, but moving in the opposite direction. Hence, the center of the eccentric must be at E vertically below C, and must have passed through the angle $C\,O\,E = 2\;C\,O\,B = 2 \times 72° \; 32'' = 145° \; 4'$. Therefore, the crank has moved 145° 4′ from the dead-center position, and now makes an angle of $\frac{145° \; 4'}{2} = 72° \; 32'$, with $A\,B$, the diameter through the dead points.

(663) (*a*) Efficiency $= \frac{12.325}{15.36} = .8024 = 80.24\%$. Ans.

(*b*) Area of piston $= .7854 \times 9^2 = 63.617$ sq. in.

Stroke $= 12'' = 1$ foot.

Using formula **98**, I.H.P. $= \frac{PLAN}{33,000}$, or

$$P = \frac{\text{I.H.P.} \times 33,000}{LAN} = \frac{15.36 \times 33,000}{1 \times 63.617 \times 240 \times 2} = 16.6 \text{ lb.}$$

(664) See Arts. **1223** and **1224.**

(665) The construction is shown in Fig. 54. Choose a suitable length for the diagram, say 3 inches. Draw the atmospheric line $M\,N$. Choose a scale of pressures, say 40 pounds per inch, and draw the vacuum line $O\,X$ parallel to $M\,N$ and $\frac{14.7}{40} = .3675''$ below it. On $O\,X$ lay off the length of the diagram $A\,B = 3''$. This length represents to some scale the volume of the cylinder, and 7% of this length must then represent to the same scale the clearance. Therefore, from A, lay off $A\ O$ equal to 7% of $A\ B = 3 \times .07 = .21''$, and from O draw $O\ Y$ perpendicular to $O\ X$. Lay off $M\,C = \frac{70}{40} = 1\frac{3}{4}''$ to represent the boiler pressure, and through C draw $C\,E$ parallel to $O\,B$. The number of expansions is 3. Hence, lay off $C\,D = \frac{1}{3}\,O\,B$, and D will be the point of cut-off. Through D, draw the equilateral hyperbola $D\,F$, as explained in Art. **1161**, taking O as the point from which to draw the radial lines. $D\,F$ is the expansion line. The back pressure is 2 lb. above the atmosphere. Hence, the line $G\,H$ drawn parallel to $M\,N$ and $\frac{2}{40} = \frac{1}{20}''$ above 1 will represent the back-pressure line of the diagram. Since the steam is compressed to 40 lb., the point where compression ends may be found by measuring $P\,K = \frac{40}{40} = 1''$ above $M\,N$. Through K, the equilateral hyperbola $K\,H$ is drawn in the same manner as $D\,F$. Draw $K\,L$ perpendicular to $M\,N$, and the diagram is finished.

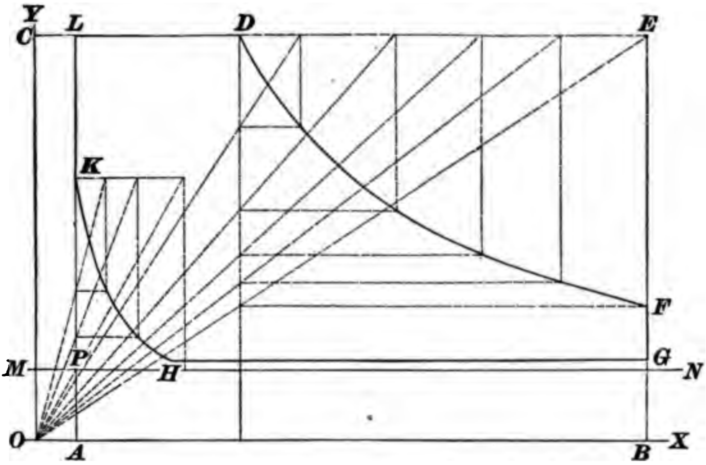

FIG. 54.

(666) See Art. **1308.**

(667) To obtain an idea of the condensation of the steam in the cylinder. There is generally more condensation at cut-off than at release, on account of reevaporation, and if the steam consumption be calculated from both points, the difference will tell more or less, approximately, the amount of condensation at cut-off.

(668) The data assumed will, of course, depend largely upon circumstances. Suppose the engine to be non-condensing, and to run with a piston speed of 600 feet per minute, and use steam at a boiler pressure of 75 lb. The mechanical efficiency will probably be about 80%, which requires the engine to develop $\frac{240}{.80} = 300$ I. H. P. Assuming the number of expansions to be $2\frac{1}{2}$, the probable M. E. P. is, from formula **99**, M. E. P. $= \frac{.9\,P(1 + 2.3 \log e)}{e} - .9\,p = \frac{.9 \times 89.7\,(1 + 2.3 \log 2.5)}{2.5} - .9 \times 17 = 46.55.$

Now, using formula **98**, I. H. P. $= \frac{PLAN}{33,000}$, or $A = \frac{33,000 \text{ I. H. P.}}{PLN}$. But $LN =$ piston speed $= 600$ ft.

Hence, $A = \frac{33,000 \times 300}{46.55 \times 600} = 354.46$ sq. in. = area of piston.

The diameter corresponding to this area is $21\frac{1}{4}''$.

For this diameter a fair stroke would be 30″, the number of revolutions being from formula **100**,

$$R = \frac{6\,S}{L} = \frac{6 \times 600}{30} = 120.$$

The engine would be made $21\frac{1}{4}'' \times 30''$, and run at 120 rev. per min. Ans.

(669) The scale of the spring should be in general not less than $\frac{1}{2}$ of the boiler pressure.

(*a*) $\frac{54}{2} = 27$; hence, a 30 spring should be used.

(*b*) $\frac{115}{2} = 57\frac{1}{2}$; hence, a 60 spring should be used.

(*c*) $1.834'' \times 40 = 73.36$ lb. per sq. in. Ans.

(670) Weight of condensing water per pound of steam $= \frac{13,580}{906}$.

Total heat of steam at an absolute pressure of 7 pounds is, from the table, 1,135.908. Substituting in formula **105,**

$W = \frac{H - t_2 + 32}{t_1 - t_2}$, or $t_2 = H + 32 - W(t_1 - t_2)$, we obtain

$t_2 = 1,135.908 + 32 - \frac{13,580}{906}(120 - 52) = 148.66°$. Ans.

(671) The areas of the three pistons are proportional to the squares of their respective diameters. Hence, the M. E. P. of the high-pressure cylinder reduced to the low-pressure cylinder is $72 \times \frac{27^2}{66^2} = 12.05$ lb. per sq. in. The M. E. P. of the intermediate cylinder reduced to the high-pressure cylinder is $40 \times \frac{42^2}{66^2} = 16.2$ lb. per sq. in.

The total M. E. P. reduced to the low-pressure cylinder is, consequently, $12.05 + 16.2 + 16.5 = 44.75$ lb. per sq. in. Area of low-pressure piston $= .7854 \times 66^2 = 3,421.2$ sq. in.

(*a*) Substituting, now, in formula **98,**

$$\text{I. H. P.} = \frac{PLAN}{33,000} = \frac{44.75 \times 4 \times 3,421.2 \times 70 \times 2}{33,000} = 2,598.$$ Ans.

(*b*) The work done in each cylinder is proportional to the M. E. P. of the cylinder reduced to the low-pressure cylinder. Hence, the percentage of the total work done in the high-pressure cylinder is $\frac{12.05}{44.75} = .269 = 26.9\%$. Ans. The percentage of the work done in the intermediate cylinder is $\frac{16.2}{44.75} = .362 = 36.2\%$. Ans. Lastly, the percentage of the work done in the low-pressure cylinder is $\frac{16.5}{44.75} = .369 = 36.9\%$. Ans.

(672) See Art. **1287.**

(673) The number of expansions is determined from formulas **97** and **96,**

$$k = \frac{k_1 + i}{1 + i} = \frac{.25 + .025}{1 + .025} = \frac{.275}{1.025}.$$

$$e = \frac{1}{k} = \frac{1}{\frac{.275}{1.025}} = \frac{1.025}{.275} = 3.727.$$

(*a*) The M. E. P. may now be found from formula **99.**

$$\text{M. E. P.} = \frac{.9\,P(1 + 2.3 \log e)}{e} - .9p =$$

$$\frac{.9 \times 98.7\,(1 + 2.3 \log 3.727)}{3.727} - .9 \times 3 = 52.455 \text{ lb. per sq. in.}$$
Ans.

(*b*) Using formula **98,** $\text{I. H. P.} = \frac{PLAN}{33{,}000}$, or $A = \frac{33{,}000 \times \text{I. H. P.}}{PLN} = \frac{33{,}000 \times 120}{52.455 \times 500} = 151$ sq. in. The required diameter is, therefore, $\sqrt{\frac{151}{.7854}} = 13\frac{7}{8}''$. Ans.

(674) Since the gauge pressure is 93 lb., the absolute pressure is 93 + 14.7 = 107.7 lb. Substituting the logarithm of 107.7 in formula **90,**

$$2.03222 = 6.1007 - \frac{2{,}719.78}{T} - \frac{400{,}215}{T^2}.$$

Clearing of fractions and transposing,

$$4.06848\,T^2 - 2{,}719.78\,T = 400{,}215.$$

As this is an affected quadratic, it may be solved by the rule given in algebra; hence, dividing by the coefficient of T^2, $T^2 - 668.5\,T = 98{,}369.66$. Completing the square $T^2 - 668.5\,T + 111{,}723.2 = 210{,}092.86$. Extracting square root, $T - 334.25 = 458.359$, or $T = 792.609°$. Therefore, $792.609 - 460 = 332.609°$. Ans.

(675) See rule in Art. **1246,** and the answer to question 662.

(676) The volume discharged in 1 stroke is

$$\frac{18^2 \times .7854 \times 24}{1{,}728} \text{ cu. ft.,}$$

and the volume discharged in 1 minute is

$$\frac{18^2 \times .7854 \times 24 \times 175 \times 2}{1{,}728}.$$

Formula **93** gives the work in foot-pounds which is done in 1 stroke; hence, if multiplied by the number of strokes per minute and divided by 33,000, the result will be the horsepower. The pressure is 62.4 lb. Substituting these values of the pressure and volume, in formula **93**, and dividing by 33,000, we have

$$\frac{144 \times 62.4 \times 18^2 \times .7854 \times 24 \times 175 \times 2}{1{,}728 \times 33{,}000} = 336.825 \text{ horsepower. Ans.}$$

(677) Dividing the diagram into 10 equal parts, and measuring the ordinates drawn at the middle of these divisions, the sum of their lengths for diagram *A* is 7.16″, for diagram *B*, 7.24″, and for diagrams *C* and *D*, 4.68″ and 4.72″, respectively. Dividing these sums for *A* and *B* by 10 and multiplying by 60, the scale of spring, we have,

for *A*, $7.16 \times \frac{60}{10} = 42.96$ lb. per sq. in.;

for *B*, $7.24 \times \frac{60}{10} = 43.44$ lb. per sq. in.

Adding and dividing by 2, the mean effective pressure for the high-pressure cylinder is found to be $\frac{42.96 + 43.44}{2} = 43.2$ lb. per sq. in.

The M. E. P. for the low-pressure cylinder is,

for *C*, $4.68 \times \frac{30}{10} = 14.04$ lb. per sq. in.;

for *D*, $4.72 \times \frac{30}{10} = 14.16$ lb. per sq. in.

Adding and dividing by 2, the M. E. P. for the low-pressure cylinder is $\frac{14.04 + 14.16}{2} = 14.1$ lb. per sq. in.

Reducing the M. E. P. of the high-pressure cylinder to the area of the low-pressure piston, we have $43.2 \times \frac{13^2 \times .7854}{20^2 \times .7854} = 18.252$ lb. per sq. in.

Hence, $14.1 + 18.252 = 32.352$ lb. per sq. in. = the M.E.P. if the work was all done in the low-pressure cylinder.

Therefore, $\text{I. H. P.} = \frac{PLAN}{33,000} =$

$$\frac{32.352 \times \frac{15}{12} \times 20^2 \times .7854 \times 230 \times 2}{33,000} = 177.1 \text{ I. H. P. Ans.}$$

(678) Fig. 55 shows diagram *A* and diagram *D* drawn as mentioned in Art. **1305;** that is, the scale of pressures on diagram *D* has been reduced to 60, the same as on diagram *A*. In Fig. 56 the length of diagram *A* has been reduced in the proportion of the ratio of the volume of the high-pressure cylinder to that of the low-pressure cylinder. By reading Arts. **1304** and **1305,** the student should have no trouble in drawing the diagrams by aid of the above figures.

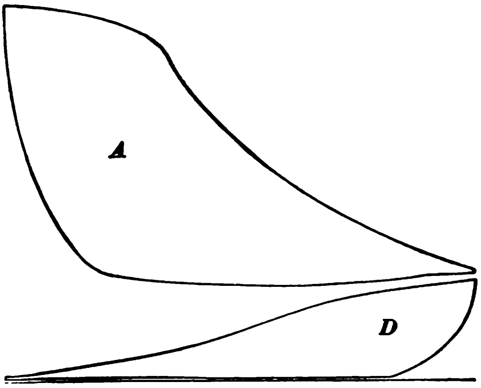

FIG. 55.

(679) Using a scale divided into thirtieths of an inch, and locating for this case a point $\frac{28''}{30}$ above the vacuum line (which should be drawn previous to this), we draw through this point a line parallel to the atmospheric line. The length of the portion of this line included between the bounding lines of the diagram is 2.95″, and the absolute pressure which it represents is 28 lb. The whole length of the diagram is $3\frac{1}{2}''$. The weight of a cubic foot of steam at this pressure is, from the table of the Properties of Saturated Steam, .069545 lb. The M. E. P. was found to be 30.75 lb. per sq. in., in example 649.

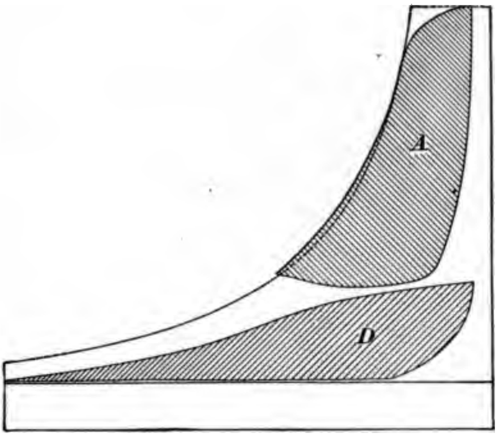

FIG. 56.

Substituting these values in formula **101**,

$Q = \dfrac{13,750 \times 2.95 \times .069545}{30.75 \times 3.5} = 26.21$ lb. per I. H. P. per hour. Ans.

(680) For (*a*) and (*b*) see Art. **1292.** (*c*) No. The stroke may be so short that the piston speed will be low, although the rotative speed is high.

(681) The absolute pressure $= 123 + 14.7 = 137.7$ lb. Logarithm of $137.7 = 2.13893$. Substituting in formula **90,**

$$2.13893 = 6.1007 - \frac{2,719.78}{T} - \frac{400,215}{T^2}.$$

Clearing of fractions and transposing, $3.96177\ T^2 - 2,719.78\ T = 400,215$.

Solving this equation for T in the same manner as was done in example 674, $T = 811.49°$, or $t = 811.49 - 460 = 351.49°$. Ans.

(682) Since, according to formula **98**,

$$\text{I. H. P.} = \frac{PLAN}{33,000},\ P = \frac{\text{I.H.P.} \times 33,000}{LAN}.$$

Substituting the values given in the present example,

$$P = \frac{300 \times 33,000}{\frac{18}{12} \times 22^2 \times .7854 \times 200 \times 2} = 43.406 \text{ lb. per sq. in.}$$ Ans.

(683) See answer to question 675.

(684) Drawing and uniting the lines $s\ t\ u$ and $v\ t\ r$ of diagrams A and B, respectively, the diagram shown at (a), Fig. 57, is drawn. (Fig. 57 is not drawn to scale.) Dividing the part $r\ s\ t$ into 10 equal parts, and measuring the ordinates drawn at the middle of those parts, the sum of their lengths is 7.78″, and the average length (mean ordinate) is $7.78 \div 10 = .778''$. The length of a line parallel to the atmospheric line included between the point t and the line $s\ r$ is 2.23″. Hence, area of $s\ t\ r = .778 \times 2.23 = 1.7349$ sq. in. Dividing $v\ t\ u$ into 5 equal parts, the sum of the middle ordinates is 1.8″. $1.8 \div 5 = .36'' =$ mean ordinate of $v\ t\ u$. Length from point t to line $v\ u = .37''$. Area of $v\ t\ u = .36 \times .37 = .1332$ sq. in. Subtracting area $v\ t\ u$ from area $r\ s\ t$, $1.7349 - .1332 = 1.6017$ sq. in. The length $r\ u = 2.6''$. Hence, the mean pressure urging the piston ahead is $\frac{1.6017 \times 60}{2.6} = 36.96$ lb. per sq. in. Ans.

(685) This example is worked like the previous examples where the horsepower is to be found from the diagrams. Dividing the length of the diagrams into ten equal parts, and measuring the middle ordinates, their sum for diagram

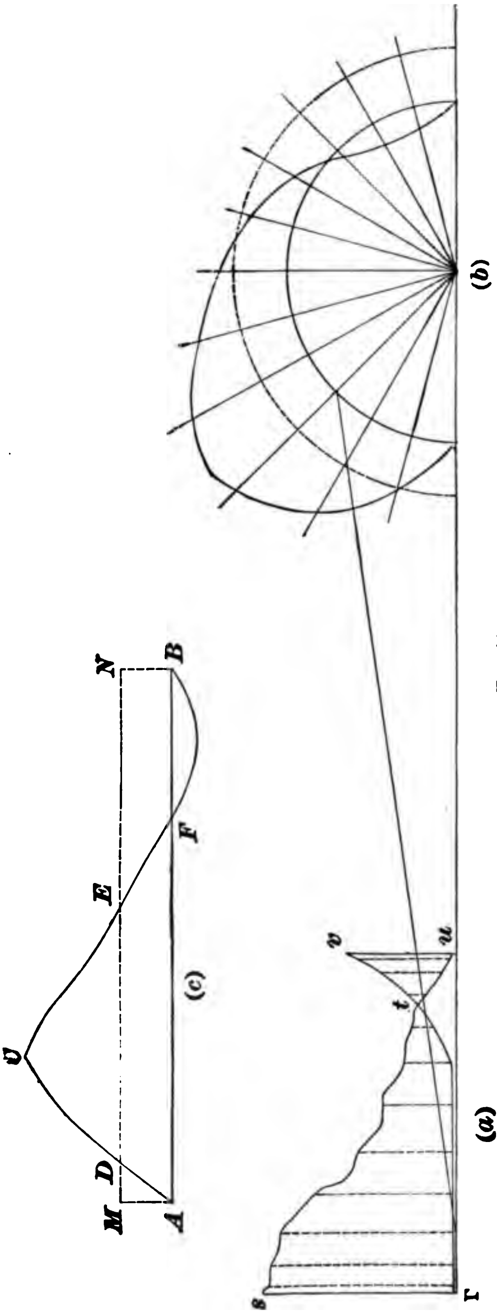

FIG. 57.

A is found to be 6.04″, and for *B*, 6.62″. The mean ordinate for $A = \frac{6.04}{10} = .604$, and for B, $\frac{6.62}{10} = .662$. Hence, the total M. E. P. $= \frac{.604 + .662}{2} \times 60 = 37.98$ lb. per sq. in.

$$\text{I. H. P.} = \frac{37.98 \times 1 \times 13^2 \times .7854 \times 300 \times 2}{33,000} = 91.66. \quad \text{Ans.}$$

(686) Drawing a line parallel to the atmospheric line, and at a distance from the vacuum line corresponding to a pressure of 40 lb., absolute, the length included between the bounding lines of diagram *A* is 1.46″, and the length between the bounding lines of diagram *B* is 1.72″. The length of both diagrams is 2.6″. The weight of a cubic foot of steam at an absolute pressure of 40 lb. is .097231 lb. The M. E. P. for card $A = .604 \times 60 = 36.24$ lb., and for card *B*, $.662 \times 60 = 39.72$ lb. (See example 685.) Substituting in formula **101,** we have, for card *A*,

$$Q = \frac{13,750 \times 1.46 \times .097231}{36.24 \times 2.6} = 20.716 \text{ lb.},$$

and for card *B*,

$$Q = \frac{13,750 \times 1.72 \times .097231}{39.72 \times 2.6} = 22.267 \text{ lb.}$$

Taking the average of both cards,

$$Q = \frac{20.716 + 22.267}{2} = 21.49 \text{ lb. per I. H. P. per hour.} \quad \text{Ans.}$$

(687) In Fig. 57 is shown the solution to this question. The cut is not drawn to scale, and the student should redraw it full size. By referring to Art. **1313,** the student should have no difficulty.

Find the areas of *A C F* and *F B*; subtract the smaller from the larger, and divide by the length *A B*; the result is the height *A M*. Draw *M N* parallel to *A B*; also the dotted semicircle, at a distance from the full semicircle equal to *A M*.

(688) Compute the area of *D C E D* in the usual manner; for the present case it equals .78 sq. in. The stroke of the engine $= 12'' = 1$ ft. and the crank-pin travels in one stroke a distance $= \frac{1 \times 3.1416}{2} = 1.5708$ feet. The length $A\,B = 4.08''$. Consequently, $1''$ of length on $A\,B = \frac{1.5708}{4.08} =$ .385 ft. of crank-pin travel. Since the vertical scale of pressures is $1'' = 60$, H, in formula **107,** $= .78 \times 60 \times .385 =$ 18.018 lb. per sq. in. of piston area. Substituting in formula **107,** we have $W = \frac{A\,H\,g}{n^2\,E\,v_0^2}$

$$= \frac{13^2 \times .7854 \times 18.018 \times 32.16}{5^2 \times \frac{1}{50} \times \left(\frac{300 \times 2 \times 1 \times 3.1416}{2 \times 60}\right)^2} = 623 \text{ lb.} \quad \textbf{Ans.}$$

PRINCIPLES
OF
REFRIGERATION.

(QUESTIONS 689–748.)

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(689) The friction head corresponding to a pressure of 5 pounds per square inch is 5 ÷ .43 = 11.6 feet. With this friction head a 1-inch pipe 347 feet long, according to formula **125**, will carry

$$Q = 28.5\sqrt{\frac{11.6}{347}} = 5.21 \text{ gallons.}$$

The actual quantity required is 320 gallons; hence the factor for the pipe is 320 ÷ 5.21 = 61. This lies between 40.5 for 4-inch and 72 for 5-inch pipe, and the larger size should be used.

(690) See Art. **1441.**

(691) See Art. **1450.**

(692) See Art. **1422.**

(693) See Art. **1421.**

(694) From formula **115**,

$$t = \frac{2{,}196}{6.2495 - \log p} - 460.66 =$$

$$\frac{2{,}196}{5.08218} - 460.66 = -28.562.$$

From formula **116**,

$$v'_1 = \frac{.016}{.6502 + .000778 \times 28.562} = .02379.$$

In this solution we *add* the product $.000778 \times 28.562$ because the temperature is *negative*, that is, below 0° F.

From formula **118**,

$$r = 555.5 + .613 \times 28.562 - .000219 \times \overline{28.562}^2 = 572.83.$$

From formula **117**,

$$v = .02379 - .00107 \times \frac{572.83}{14.7} \times 28.562 + .4923 \times \frac{572.83}{14.7}$$
$$= 18.018.$$

$$w' = \frac{1}{v'} = \frac{1}{18.018} = .0555.$$

Collecting the results, we have

$$\left.\begin{array}{l} t = -28.562^\circ \text{ F.} \\ v'_1 = .02379 \text{ cu. ft.} \\ v = 18.018 \text{ cu. ft.} \\ w = .0555 \text{ lb.} \\ r = 572.83 \text{ B. T. U.} \end{array}\right\} \text{Ans.}$$

(695) The absolute temperature is 230° + 460° = 690°, and the absolute pressure is 114.7 pounds per square inch.

From formula **119**,

$$v = \frac{.62\,M\,T}{p} = \frac{.62 \times 1 \times 690}{114.7} = 3.73 \text{ cu. ft.} \quad \text{Ans.}$$

(696) See Art. **1364.**

(697) (*a*) See Arts. **1352, 1353, 1427,** and **1357.**

(*b*) The principal advantage of ammonia is its comparatively great latent heat. See Art. **1358.**

(698) See Art. **1429.**

(699) (*a*) and (*b*) See Art. **1350.**

(700) The heat abstracted per hour is

$$Q_2 = 142.65 \times 250 = 35{,}662.5 \text{ B. T. U.};$$

therefore, the heat abstracted per stroke is

$$\frac{35{,}662.5}{75 \times 60} = 7.925 \text{ B. T. U.} \quad \text{Ans.}$$

The initial temperature being 500° absolute, the absolute temperature at the end of compression is

$$T_b = T_a\left(\frac{p_2}{p_1}\right)^{.29078} = 500 \times \left(\frac{44.7}{14.7}\right)^{.29078} = 690.9^\circ.$$

The absolute temperature at the end of expansion is

$$T_d = T_c\frac{T_a}{T_b} = 555 \times \frac{500}{690.9} = 401.65^\circ;$$

hence, $t_d = 401.65^\circ - 460^\circ = -58.35^\circ.$

One pound of air in passing from -58.35° to 40° will absorb $.23751 \times [40 - (-58.35)] = 23.359$ B. T. U.; hence the weight of air required per stroke is

$$7.925 \div 23.359 = .3393 \text{ lb.} \quad \text{Ans.}$$

The volume of the air entering the compressor per stroke is

$$V_a = \frac{RMT_a}{p_1} = \frac{.37052 \times .3393 \times 500}{14.7} = 4.276 \text{ cu. ft.} \quad \text{Ans.}$$

The volume of the expansion cylinder is

$$V_d = V_a\frac{T_d}{T_a} = 4.276 \times \frac{401.65}{500} = 3.435 \text{ cu. ft.} \quad \text{Ans.}$$

The work per stroke is

$$W = Js_pM[(T_b - T_c) - (T_a - T_d)] =$$

$$778 \times .23751 \times .3393 \times [(690.9 - 555) - (500 - 401.65)]$$

$$= 2{,}354+ \text{ foot-pounds.}$$

The net horsepower required is

$$H = \frac{nW}{33{,}000} = \frac{75 \times 2{,}354}{33{,}000} = 5.35 \text{ H. P.} \quad \text{Ans.}$$

The cooling water required per minute is

$$\frac{75 \times .23751 \times .3393 \times (690.9 - 555)}{85 - 65} = 40.069 \text{ lb.}$$

The efficiency of the machine is

$$\frac{T_a}{T_b - T_a} = \frac{500}{690.9 - 500} = 2.62. \quad \text{Ans.}$$

(701) The sample *B*. See Art. **1437.**

(702) See Art. **1413.**

(703) See Art. **1438.**

(704) See Art. **1434.**

(705) Using formula **124,** $G = \frac{25\,T}{t_2 - t_1}$, or $T = \frac{G\,(t_2 - t_1)}{25}$, we have $T = \frac{247 \times 4.5}{25} = 44.46$ tons. Ans.

(706) Using formula **124,**

$$G = \frac{25 \times 40}{6.5} = 153.85 \text{ gallons. Ans.}$$

(707) See Art. **1446.**

(708) See Art. **1411.**

(709) (*a*) Using formula **124,** the quantities for the rooms are as follows:

Room *A*, $\frac{25 \times 29}{9 - 5} = 181\frac{1}{4}$ gallons.

Room *B*, $\frac{25 \times 12}{4} = 75$ gallons.

Room *C*, $\frac{25 \times 18}{4} = 112\frac{1}{2}$ gallons.

Room *D*, $\frac{25 \times 4}{4} = 25$ gallons.

(*b*) The pipe from *C* to *D* carries the 25 gallons for room *D*. The pipe from *B* to *C* carries the $112\frac{1}{2}$ gallons to *C* and also the 25 gallons to *D*, or $112\frac{1}{2} + 25 = 137\frac{1}{2}$ gallons in all. The pipe from *A* to *B* carries $25 + 112\frac{1}{2} + 75 = 212\frac{1}{2}$ gallons, and that from the engine room to *A* carries the total quantity, $25 + 112\frac{1}{2} + 75 + 181\frac{1}{4} = 393\frac{3}{4}$ gallons. With a friction head of 12 feet, the quantities flowing through a 1-inch pipe for the various lengths are as follows:

From engine room to room *A*, $28.5\sqrt{\frac{12}{60}} = 12.7$ gal.

From room *A* to room *B*, $28.5\sqrt{\frac{12}{30}} = 18$ gal.

From room *B* to room *C*, $28.5\sqrt{\frac{12}{75}} = 11.4$ gal.

From room *C* to room *D*, $28.5\sqrt{\frac{12}{40}} = 15.6$ gal.

		Diameter of pipe.
Factor for pipe to room *A*, $393.7 \div 12.7 = 31$.	Ans.	4-inch.
Factor for pipe to room *B*, $212.5 \div 18 = 11.8$.		3-inch.
Factor for pipe to room *C*, $137.5 \div 11.4 = 12.1$.		3-inch.
Factor for pipe to room *D*, $25 \div 15.6 = 1.6$.		1¼-inch.

(710) See Art. **1443.**

(711) At the given temperatures, 20° and 85°, the properties of saturated ammonia are as follows:

Temp.	Absolute Temp.	Pressure.	Latent Heat.	Volume per pound.
20°	480°	47.95	543.15	5.84
85°	545°	167.86	501.81	1.77

To find the temperature of the superheated gas at the end of compression, we have

$$T_b = T_a\left(\frac{p_2}{p_1}\right)^{.2308} = 480 \times \left(\frac{167.86}{47.95}\right)^{.2308} = 640.96^\circ.$$

$$t_b = 640.96^\circ - 460^\circ = 180.96^\circ.$$

From the formula $F = .00505\, n\, M\,[r_1 - (t_c - t_a)]$,

$$M = \frac{F}{.00505\, n\,[r_1 - (t_c - t_a)]} = \frac{12}{.00505 \times 70 \times [543.15 - (85 - 20)]} =$$

.071 lb., the weight of ammonia circulated per stroke.

The volume of 1 pound at the given temperature is 5.84 cu. ft.; hence, the theoretical volume of the compressor cylinder is

$5.84 \times .071 = .415$ cu. ft. Ans.

To find the actual dimensions, add $\frac{1}{8}$ to the volume a make the stroke double the diameter.

For the horsepower, we have

$$H = \frac{n J M \{r_2 - r_1 + .508\,(t_b - t_c) + t_c - t_a\}}{33{,}000} =$$

$$\frac{70 \times 778 \times .071 \times \{501.81 - 543.15 + .508(180.96 - 85) + 85 - 20\}}{33{,}000} =$$

8.48 horsepower. Ans.

The horsepower of the steam cylinder should be $\frac{1}{3}$ grea or $8.48 \times \frac{4}{3} = 11.3$.

The heat given up to the cooling water per stroke is

$$Q_1 = M[.508\,(t_b - t_c) + r_2] =$$

$$.071 \times [.508\,(180.96 - 85) + 501.81] = 39.09 \text{ B. T. U.}$$

The cooling water required per minute is therefore

$$G = \frac{n\,Q_1}{t_f - t_e} = \frac{70 \times 39.09}{80 - 60} = 136.81 \text{ pounds.} \quad \text{Ans.}$$

The theoretical efficiency is

$$E = \frac{r_1 - (t_c - t_a)}{r_2 - r_1 + .508\,(t_b - t_c) + t_c - t_a} = \frac{478.15}{72.408} = 6.6. \quad \text{A}$$

(712) See Art. **1375.**

(713) See Art. **1376.**

(714) (*a*) See Art. **1384.**

(*b*) See Art. **1385.**

(*c*) See Art. **1405.**

(715) (*a*) and (*b*) See Art. **1380.**

(716) The absolute pressures are 194.7 and 24.7 pou respectively, and the corresponding temperatures, Table are 94° and — 8.5°, nearly. The latent heat at the lo temperature is about 560.7 B.T.U. and the weight of a c foot is .0911 lb.

(*a*) To find the capacity, we use formula **126,** $.00505\, n\, C w\, [r_1 - (t_c - t_a)]$. In the present case, the prod

$n\,C$ is 357 cubic feet, the cylinder displacement per minute. Substituting,

$$F = .00505 \times 357 \times .0911 \times [560.7 - (94 - (-8.5))] =$$

75.254 tons. Ans.

(*b*) $75.254 \times (100 - .30) = 52.68$ tons. Ans.

(717) See Art. **1360.**

(718) See Art. **1366.** (*a*) It is stated in this article that for the same piston displacement, the ammonia machine gives a cooling effect of 62.75 B. T. U. and a sulphur-dioxide machine gives a cooling effect of 22.88; hence for the same cooling effect, the volume of the sulphur-dioxide cylinder must be

$$2 \text{ cu. ft.} \times \frac{62.75}{22.28} = 5.63, \text{ or } 5\tfrac{5}{8} \text{ cu. ft., nearly. Ans.}$$

(*b*) Similarly, the volume of the ether cylinder would be

$$2 \text{ cu. ft.} \times \frac{62.75}{3.68} = 34 \text{ cu. ft., nearly. Ans.}$$

(*c*) For the carbon-dioxide machine, the cylinder volume would be

$$2 \text{ cu. ft.} \times \frac{62.75}{248.18} = .5 \text{ cu. ft., nearly. Ans.}$$

(719) (*a*) See Art. **1369.**
(*b*) See Art. **1428.**

(720) See Art. **1368.**

(721) (*a*) and (*b*) See Art. **1332.**

(722) The volume of the expansion cylinder is

$$.7854 \times \frac{15}{12} = .982 \text{ cu. ft.}$$

The absolute temperature of the cold room is $460° + 30° = 490°$, and that of the air at the end of expansion is $460° - 43° = 417°$. Substituting known values in formula **114,**

$$F = .003235 \times 100 \times 14.7 \times .982 \times \left(\frac{490}{417} - 1\right) =$$

.817 ton in 24 hours. Ans.

(723) See Art. **1348.**

(724) See Art. **1342.**

(725) (*a*) and (*b*) See Art. **1340.**

(726) (*a*) and (*b*) See Art. **1338.**
(*c*) See Art. **1339.**

(727) Referring to Tables 28 and 29, it is evident that calcium brine must be used, and that the specific gravity should be about 1.21. Ans.

(728) See Arts. **1436** and **1437.**

(729) See Arts. **1432** and **1433.**

(730) (*a*) See Art. **1430.**
(*b*) and (*c*) See Art. **1431.**

(731) See Art. **1418.**

(732) See Arts. **1427** and **1428.**

(733) See Art. **1421.**

(734) See Art. **1411.**

(735) See Arts. **1412** and **1416.**

(736) Using formula **110,**

$$F = \frac{635{,}342 \div 6\frac{1}{3}}{11{,}887.5} = \frac{635{,}342 \times 3}{11{,}887.5 \times 19} = 8.44 \text{ tons.} \quad \text{Ans.}$$

(737) See Art. **1332,** *et seq.*

(738) (*a*) See Art. **1333.**
(*b*) See Art. **1335.**

(739) See Art. **1336.**

(740) See Art. **1337.**

(741) The temperatures corresponding to the gauge pressures 135 lb. and 25 lb. are, respectively, 78.3° and 11.3°, and the latent heat corresponding to the gauge pressure of 25 lb. is about 548.5. See Table 26.

The quantity of liquid used per hour is 113 cu. ft. $\div 2\frac{1}{5} =$ 51.36 cu. ft. According to Table 26, a pound of ammonia liquid at a pressure of 25 lb. per sq. in. has a volume of about .025 cu. ft.; hence a cubic foot will weigh about $\frac{1}{.025} = 40$ lb. and the weight of liquid used per hour is (40×51.36) lb.

Substituting in formula **127**,

$$F = .000084\,P[r - (t_c - t_a)] =$$
$$.000084 \times 40 \times 51.36 \times [548.5 - (78.3 - 11.3)] =$$
$$83.09 \text{ tons. Ans.}$$

(742) The piston displacement is $.7854 \times 6^2 \times 10\frac{1}{2} =$ 296.84 cu. in. The volume of brine circulated per hour is, therefore, $\frac{296.84 \times 82 \times 60}{1{,}728}$ cu. ft.

Since the specific gravity of the brine is 1.2, the weight of a cubic foot is (62.42×1.2) lb.; hence the weight of brine circulated per hour is $\frac{296.84 \times 82 \times 60 \times 62.42 \times 1.2}{1{,}728}$ lb. = 63,307 lb. The heat abstracted by the brine per hour is $.75 \times 63{,}307 \times (15 - 11)$ (see Art. **1456**), and according to formula **110**,

$$F = \frac{h}{11{,}887.5} = \frac{.75 \times 63{,}307 \times (15 - 11)}{11{,}887.5} = 15.98 \text{ tons.}$$

Deducting 20 per cent., the net capacity is

$$15.98 \times .8 = 12.78 \text{ tons. Ans.}$$

(743) (*a*) See Art. **1466.**
(*b*) See Art. **1467.**

(744) See Art. **1460.**

(745) (*a*) See Arts. **1454** and **1455.**
(*b*) See Art. **1456.**

REFRIGERATING
AND
ICE-MAKING MACHINERY.

(QUESTIONS 749–794.)

---

(749) See Arts. **1506** and **1507.**

(750) See Art. **1492.**

(751) To apply formula **131,** we have $g = 1.06$, $s = .892$, $b = 327$, $T - t = 68^\circ - 40^\circ = 28^\circ$. Substituting, we get

$$F = .0009197 \times 1.06 \times 327 \times .892 \times 28 = 7.96 \text{ tons.}$$

This assumes the cooling to take place in 24 hours. As the time is to be only 4 hours, the capacity must be 6 × 7.96 tons = 47.76 tons. Ans.

(752) See Art. **1595.**

(753) (*a*) See Art. **1500.**
(*b*) See Art. **1502.**

(754) (*a*) and (*b*) See Art. **1478.**
(*c*) See Art. **1479.**

(755) See Arts. **1547** to **1550.**

(756) (*a*) and (*b*) See Art. **1598.**

(757) (*a*) According to Art. **1559,** 1 lineal foot of 2-inch direct-expansion pipe is necessary for every 13 cubic feet of space. Therefore, the total pipe required in the present case is $\frac{15000}{13} = 1{,}154$ ft. Ans.

(*b*) According to the same article, 1 lineal foot of $1\frac{1}{4}$-inch brine pipe is required for every 5 cubic feet of space. Therefore, length of pipe required $= \frac{15000}{5} = 3,000$ feet.
Ans.

(758) (*a*) See Art. **1588.**

(*b*) See Art. **1591.**

(759) (*a*) Total space $= 10,000 + 15,000 + 50,000 = 75,000$ cu. ft. From Art. **1544,** 1 ton of refrigeration may be allowed for the cooling of every 6,000 cubic feet. Therefore, the refrigeration required for this purpose $= \frac{75000}{6000} = 12.5$ tons. To apply formula **141,** we have $n_1 = 120$, $w_1 = 240$, $s_1 = .6$ (see Table 34), $n_2 = 50$, $w_2 = 700$, $s_2 = .8$. These values in the formula give

$$F = .0002103\,(120 \times 240 \times .6 + 50 \times 700 \times .8) = 9.52 \text{ tons.}$$

Therefore, total refrigeration required $= 12.5 + 9.52 = 22$ tons, nearly. Ans.

(*b*) According to Arts. **1559** and **1560,** 1 lineal foot of 2-inch pipe is required for every 13 cubic feet of chill room and for every 40 cubic feet of storage and pickling room. Therefore,

pipe required in chill room $= \frac{10000}{13} = 770$ ft., nearly. Ans.
pipe required in storage and pickling room =

$$\frac{50,000 + 15,000}{40} = 1,625 \text{ ft.} \quad \text{Ans.}$$

(760) See Art. **1589.**

(761) See Art. **1550.**

(762) See Table 30. 16° Beaumé. Ans.

(763) See Art. **1485.**

(764) (*a*) See Art. **1571.**

(*b*) See Art. **1574.**

(765) See Art. **1580.**

(766) (*a*) See Art. **1552.**
(*b*) See Art. **1553.**
(*c*) See Art. **1554.**

(767) (*a*) See Art. **1526.**
(*b*) See Art. **1527.**

(768) (*a*) See Art. **1481.**
(*b*) See Art. **1482.**

(769) See Art. **1570.** The gross weight of the ice, or its weight before any of it melts, is $2.25 \times 57.2 = 129$ lb., nearly. The net weight is, therefore, $129 \text{ lb.} - .05 \times 129 \text{ lb.} = 123$ lb., nearly. Ans.

(770) See Art. **1549.**

(771) See Art. **1559.**

(772) The total space is $8,000 + 20,000 + 60,000 = 88,000$ cu. ft. For a house of this size, 1 ton of refrigeration may be allowed to cool 8,000 cubic feet. Therefore, the refrigeration required for cooling the rooms is $\frac{88000}{8000} =$ 11 tons. This leaves $25 - 11 = 14$ tons for cooling the meat. From formula **141** we get, when there is only one kind of meat $\left(\text{remembering that } .000003505 = \frac{1}{285,300}\right)$,

$$F = \frac{60\, n_1\, w_1\, s_1}{285,300} = \frac{n_1\, w_1\, s_1}{4,755}, \text{ whence } n_1 = \frac{4,755\, F}{w_1\, s_1}.$$

In the present case, $F = 14$, $w_1 = 250$, $s_1 = .6$. Therefore,

$$n_1 = \frac{4,755 \times 14}{250 \times .6} = 444 \text{ hogs, nearly. Ans.}$$

(773) See Art. **1532.**

(774) (*a*) and (*b*) See Art. **1484.**

(775) The approximate formula **132** gives for the refrigeration required to cool the wort,

$$F = \tfrac{100}{38} = 2.63 \text{ tons.}$$

According to Art. **1529,** 1 ton of refrigeration will suffice for the heat of fermentation of 36 barrels; hence, for 100 barrels, $\frac{100}{36} = 2.78$ tons are needed. For cooling the cellars $\frac{60,000}{7,000} = 8.57$ tons are required. Therefore, the total refrigeration required is 2.63 tons + 2.78 tons + 8.57 tons = 13.98 tons, or 14 tons, nearly. Ans.

(776) See Arts. **1562** and **1563.**

(777) (*a*) See Art. **1586.**
(*b*) See Art. **1584.**

(778) (*a*), (*b*), and (*c*) See Art. **1562.**

(779) (*a*) See Art. **1493.**
(*b*) See Art. **1494.**

(780) See Art. **1538.**

(781) Volume of cylinder $= (3\frac{1}{8})^2 \times \frac{3.1416}{4} \times 10 = 76.7$ cu. in. The number of strokes per minute being 60, the ammonia pumped per minute is $76.7 \times 60 = 4,602$ cu. in., or $\frac{4602}{231} = 20$ gallons, nearly. As every half a gallon requires a ton of refrigeration, or every gallon 2 tons (Art. **1512**), the refrigeration required is $20 \times 2 = 40$ tons. Ans.

(782) (*a*) See Art. **1533.**
(*b*) See Art. **1534.**

(783) (*a*) and (*b*) See Art. **1537.**

(784) (*a*) and (*b*) See Art. **1516.**

(785) (*a*) and (*b*) See Art. **1540.**
(*c*) See Art. **1541.**

(786) (*a*) and (*b*) See Art. **1502.**

(787) See Art. **1486.**

(788) (*a*) Using formula **129,**

$s = 1.053 + .00015\,(60 - 39) = 1.0562.$ Ans.

(*b*) Using the same formula,

$s = 1.053 - .00015\,(73 - 60) = 1.0511.$ Ans.

(789) (*a*) and (*b*) See Art. **1496.**

(790) (*a*), (*b*), (*c*), and (*d*) See Art. **1512.**

(791) (*a*) and (*b*) See Art. **1480.**

(792) (*a*) and (*b*) See Art. **1514.**

(793) (*a*) See Art. **1509.**
(*b*) See Art. **1510.**

(794) (*a*) See Art. **1487.**
(*b*) See Art. **1488.**

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